

**On Exercises in Futility:**  
*A Glide Path Designer's View*

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*SUMMARY*

*Numerous papers on the subject of optimal glide path selection rely on heuristic approaches that do not produce conclusive results. This paper highlights the key features of the conventional framework and sketches out of a better framework (we call this framework "Glide Path 2.0").*

In recent years, academics and practitioners have been increasingly endeavoring to design glide paths that generate better outcomes. Yet, these endeavors have mostly produced raging debates and confusion. The theory of optimal glide path design is still in its infancy. Most glide paths employed by a multitude of investors are likely to generate sub-optimal outcomes.

One of the key reasons for this regrettable status-quo is that the conceptual foundation of most glide path design frameworks are quite shaky. Too often, the key concepts in these frameworks are poorly defined or not defined at all. Too often, glide path designers rely on what the late great Peter Bernstein called “rules of thumb and folklore.” Too often, casual observations substitute for rigorous demonstrations.

To be valuable, financial frameworks use assumptions and measurements that incorporate certain selected aspects of financial markets and human behavior. The value of such frameworks critically depends on the reasonability of these selections. Better frameworks strive to utilize reasonable assumptions for a simple reason: to analyze something that will not happen is an exercise in futility.

This paper discusses certain aspects of the conventional framework currently employed by most glide path designers (we call this framework “Glide Path 1.0”). Due to this framework’s shaky foundation, the framework appears incapable of producing definite conclusions. This paper discusses the structural features of Glide Path 1.0 that lead to sub-optimal outcomes.

To make this paper more focused, we tell a story of one of the better papers written within Glide Path 1.0 – Pfau-Kitces (2014). Pfau-Kitces (2014) argues that, contrary to the current industry practices, increasing equity glide paths in retirement produce better outcomes. Yet, the industry has largely ignored the conclusions of Pfau-Kitces (2014). We discuss why this *should be* the case and why this *is* the case.

The main message of this paper is *Glide Path 1.0 is inefficient and insufficient*. We also sketch out the directions of an upgrade from Glide Path 1.0 to 2.0.

### **Optimal Glide Paths: “Will-Do” Glide Path Assumption**

Pfau-Kitces (2014) analyzes the sustainability of constant inflation-adjusted payout strategies for a just-retired investor. The investor utilizes two asset

classes – stocks and bonds. There are 11 starting and 11 ending stock allocations – from 0% and 100% in 10 percentage point increments. The stock allocations adjust annually in a straight line between the starting and ending allocations. Consequently, the starting and ending stock allocations fully determine the resulting 121 glide paths. The paper discusses several outcome measurements, several sets of capital market assumptions, and several time horizons. All scenarios are analyzed using 10,000 Monte Carlo simulations.

We should emphasize one particularly important (yet hidden) assumption. In this approach, it is assumed that the investor will always dutifully follow the glide path designed today. We call such glide paths “*will-do*” glide paths. We accept the “will-do” glide path assumption in this section. We remove this assumption in the next section.

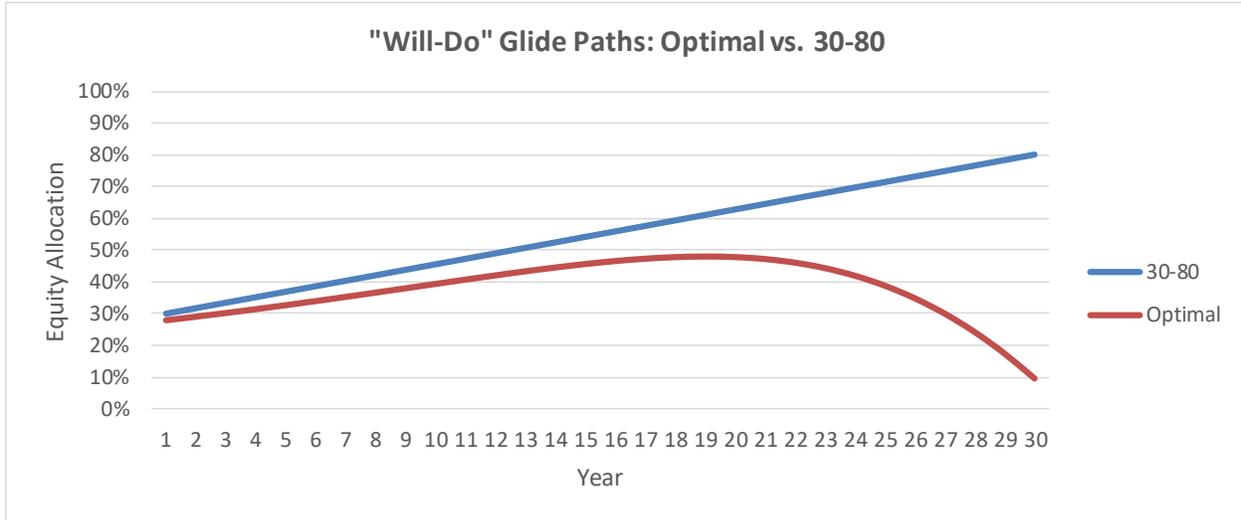
Pfau-Kitces (2014) demonstrates that different “linear” glide paths optimize different measurements under different scenarios. Most of these optimal glide paths have rising equity allocations. This finding is highlighted in the title of Pfau-Kitces (2014) – “Reducing Retirement Risk with a Rising Equity Glidepath.”

Let us perform a similar analysis in a framework that we call Glide Path 2.0. Glide Path 2.0 employs simulation-free analytical tools, i.e. the risks and uncertainties of funding problems are analyzed without simulations. The results of simulation-free stochastic analysis are transparent and replicable. In Glide Path 2.0, *optimal glide paths are outputs generated by an optimizer*, so there is no need for pre-selected glide paths.

For the purposes of this paper, we consider the success rate for a 4% withdrawal rate as the outcome measurement, the “Evensky capital market assumptions” (see Appendix for more details) and a 30-year retirement. According to Pfau-Kitces (2014), the 30-80 “linear” glide path (starting at 30% and ending at 80% of equity allocations) produces the highest success rate.

Now let us compare the 30-80 glide path to the optimizer generated glide path that produces the highest success rate (see Appendix for more details about the success rate estimate used in these calculations). The success rate generated by the optimal glide path is 74.5%; its counterpart for the 30-80 glide path is 73.3%. *Exhibit 1* presents the equity allocations of these glide paths.

**Exhibit 1**



To gauge the difference, we employ the concept of “implied surcharge” (or just “surcharge” for simplicity). The “surcharge” of glide path A (think the 30-80 glide path) over glide path B (think the optimal glide path), given an outcome measurement (think the success rate), is defined as a reduction of expected returns for glide path B that is required to make the measurement equal to its counterpart for glide path A.

In our case, the “surcharge” of the 30-80 glide path over the optimal glide path is defined as the reduction in expected returns for the optimal glide path that is required to reduce its success rate (74.5%) to its counterpart for the 30-80 glide path (73.3%). *This “surcharge” is estimated to be 7 basis points annually.*

Pfau-Kitces (2014) concludes:

*“We find, surprisingly, that rising equity glide paths in retirement ... have the potential to actually reduce both the probability of failure and the magnitude of failure for client portfolios.”*

This conclusion is correct in the sense that linear rising equity glide paths may produce better outcomes than the conventional flat glide paths. At the same time, this conclusion is incorrect the sense that the optimal glide path is not “rising equity” – its equity allocation starts at 28%, increases to 48%, then decreases to 10%. Rising equity glide paths may be sub-optimal and impose significant – and unnecessary – “surcharges” on the investor.

The authors suggest to “test a wider range of rising glidepaths”:

*“Further research might explore whether it would be better to accelerate the glidepath earlier in retirement, or alternatively to slow it in the early years and accelerate later.”*

Dear reader, do you share this author’s skepticism about limiting further research to “rising glidepaths”?

### **Optimal Glide Paths: “Expected-To-Do” Glide Path Assumption**

In the previous section, it was assumed that the investor must dutifully follow the glide path designed today throughout his 30-year time horizon (“will-do” glide path). In this section, we assume just the opposite – that the investor will revisit and reevaluate his glide path at least once at some future point. When this reevaluation takes place, let us examine the glide path the investor is expected to select (we call these glide paths “expected-to-do”).

At the reevaluation point, let us apply the methodology of Pfau-Kitces (2014). Let us consider the following funding problem:

1. At retirement, the investor has \$100 and withdraw \$4 adjusted for inflation annually.
2. At retirement, the investor’s objective is to maximize the success rate.
3. At retirement, the investor has selected the 30-80 glide path as recommended by Pfau-Kitces (2014).
4. After 15 years in retirement, the investor’s objective is still to maximize the success rate.
5. After 15 years in retirement, the investor revisits his glide path and wishes to verify the efficiency of the remaining 56-80 sub-glide path of the original 30-80 glide path.
6. After 15 years in retirement, the same capital market assumptions are used.

To be consistent with Pfau-Kitces (2014), we consider “linear” glide paths first; this assumption will be removed later in this section. We employ an optimizer that is based on simulation-free estimates of outcomes (thus, there is no need for 10% equity allocation increments).

Obviously, we do not know the accumulated asset value after 15 years in retirement. Therefore, we should look at the ranges of accumulated asset values after 15 years.

- If the asset value is less than \$30, then the highest success rate is about 15%; the optimal glide path has the starting and ending equity allocations of 100%; the 56-80 glide path is sub-optimal.
- If the asset value is greater than \$80, then the highest success rate is close to 100%; the optimal glide paths have equity allocations of less than 20%; the 56-80 glide path is sub-optimal.
- For selected asset values between \$30 and \$80, the highest success rates, corresponding linear glide paths, and “surcharges” are presented in *Exhibit 2*.

**Exhibit 2**

Assets after 15 years	\$30.00	\$40.00	\$50.00	\$55.00	\$60.00	\$70.00	\$80.00
<i>Original Glide Path</i>							
Equity Starting	56%	56%	56%	56%	56%	56%	56%
Equity Ending	80%	80%	80%	80%	80%	80%	80%
Success Rate	5.9%	26.3%	54.0%	66.3%	76.1%	88.7%	94.8%
<i>Optimal "Linear" Glide Path</i>							
Equity Starting	100%	100%	79%	28%	14%	7%	5%
Equity Ending	100%	92%	42%	46%	34%	22%	17%
Success Rate	15.1%	32.9%	54.6%	69.0%	83.6%	97.4%	99.7%
"Surcharge" (bp)	494	127	6	19	57	138	211

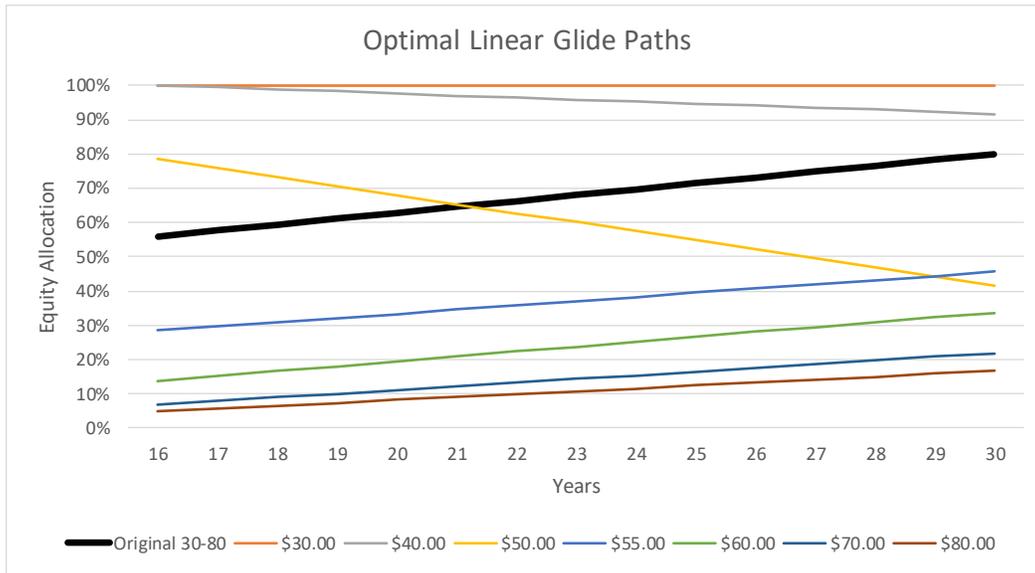
Here is how to read *Exhibit 2*. If the accumulated asset value after 15 years is \$55, then the 56-80 glide path produces a success rate of 66.3%; the optimal “linear” 28-46 glide path produces a success rate of 69.0%; the “surcharge” is 19 basis points annually, i.e. the 28-46 glide path with its returns reduced by 19 basis points and the 56-80 glide path with the original returns produce the same success rate.

*Exhibit 3* shows graphically the optimal glide paths for the selected asset values. The thick black line represents the 56-80 glide path. The other lines represent the optimal glide paths for the selected accumulated asset values.

*Exhibits 2 and 3* clearly demonstrate that the 56-80 glide path is far from being optimal for any asset value after 15 years in retirement. Since the investor is at

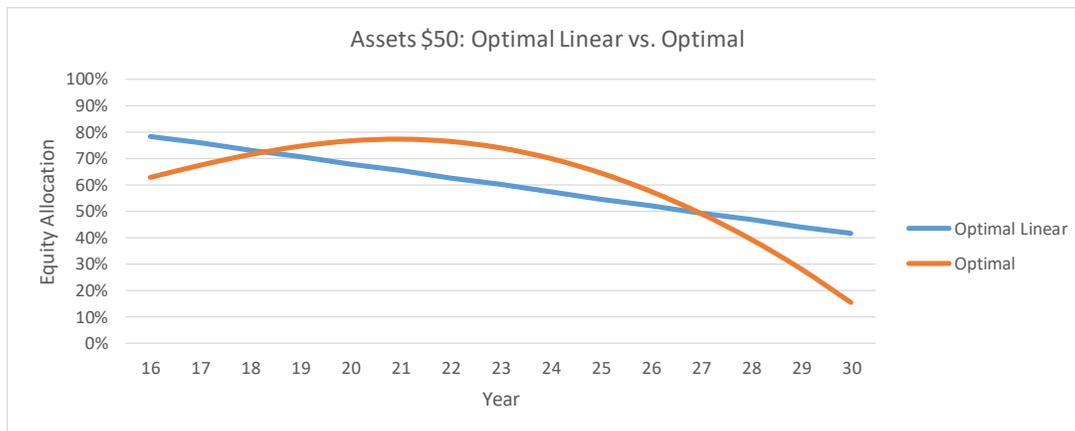
liberty to select any glide path any time, we should not rationally expect the investor to follow the original glide path after this reevaluation.

**Exhibit 3**

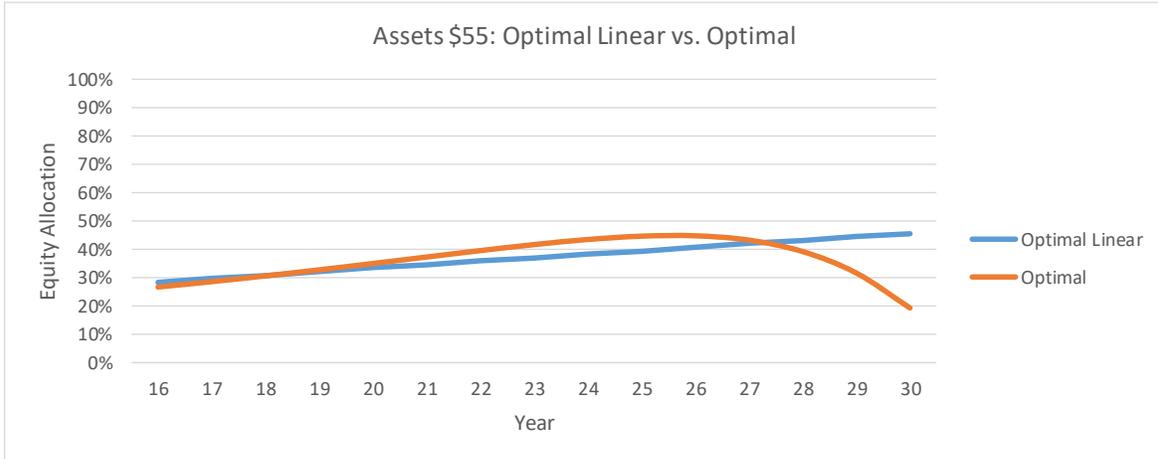


Let us remove the “linear” glide path assumption. Exhibits 4, 5, and 6 present the optimal glide paths for asset values \$50, \$55, and \$60 correspondingly.

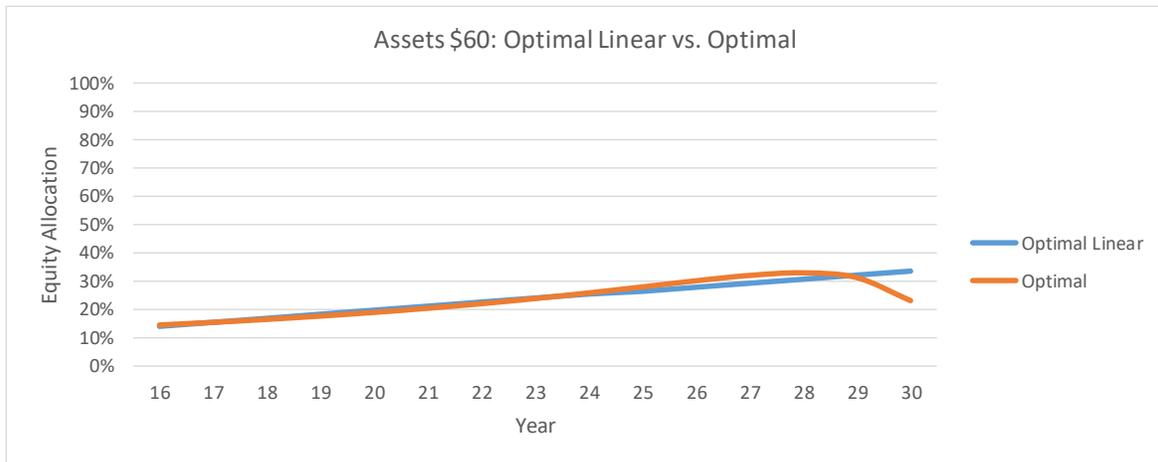
**Exhibit 4**



**Exhibit 5**



**Exhibit 6**



Dear reader, do you see any reason why the investor would continue using the remaining 56-80 segment of the original glide path?

**On Exercises in Futility**

Let us review the results of the previous sections. The 30-80 glide path generates the highest success rate among the 121 pre-selected glide paths analyzed in Pfau-Kitces (2014). However, the 30-80 glide path is in fact a sub-optimal glide path that imposes considerable “surcharges” on the investor.

One could anticipate this result from the basic features of the approach utilized in Pfau-Kitces (2014). The properties of this approach that may be theoretically suspect and put its ultimate conclusions in question include the following:

1. Simulation based estimates are inconclusive illustrations rather than definite quantitative determinations.
2. The 121 pre-selected glide paths are inputs to the model. The best glide path from a set of pre-selected glide paths may not necessarily be the best overall.
3. The “linear” equity evolution assumption may be unjustifiably restrictive.
4. The 10-percentage point equity allocation increments may be too “granular.”
5. The “will-do” glide path assumption is debatable.

Overall, our analysis of the sub-optimality of the 30-80 glide path simply confirms the shortcomings of the approach that promotes this glide path as the “best.” One could foresee this result from the limitations of Glide Path 1.0.

If we reject the “will-do” glide path assumption and accept the “expected-to-do” glide path assumption, the claim of the 30-80 glide path “optimality” is even more problematic. As demonstrated above, if the investor were to reevaluate his glide path after 15 years, he would not be expected to follow the 56-80 glide path. Analyzing a glide path that the investor would not follow makes little sense.

So, here is this story in a nutshell. Glide Path 1.0 based conventional analysis leads to either a sub-optimal glide path (under the “will-do” glide path assumption) or an “unrealizable” situation, where the investor would not follow one of the key assumptions (under the “expected-to-do” glide path assumption).

Dear reader, if you were a target date fund manager, would you enthusiastically embrace and implement the results of Pfau-Kitces (2014)?

### **From Glide Path 1.0 to 2.0: Where to Start**

The Glide Path 1.0 framework is inefficient and insufficient. It has created a lot of controversy and confusion due to its structural problems (e.g. see [Mindlin \(2016 A\)](#), [Mindlin \(2016 B\)](#), [Mindlin \(2015 D\)](#), [Mindlin \(2016 D\)](#)).

We explore the directions of an upgrade to Glide Path 2.0 in several publications (e.g. see [Mindlin \(2015 A\)](#), [Mindlin \(2015 B\)](#), [Mindlin \(2015 C\)](#)). Our approach to optimal glide path design is based on the principles of *Commitment Driven*

*Investing* (CDI); the essentials of CDI are presented in [Mindlin \(2014\)](#); the key properties of CDI glide paths are presented in [Mindlin \(2016 C\)](#).

In this article, we just sketch out the directions of an upgrade for the conceptual and analytical segments of Glide Path 1.0.

Conceptually, Glide Path 1.0 is a collection of miscellaneous observations rather than a disciplined quantitative framework. Glide Path 2.0 should be based on clearly articulated principles. These principles should include the definitions of the major stakeholders and their best interests. Glide Path 2.0 should explicitly disclose investment objectives, economic and demographic assumptions.

Glide Path 1.0 employs Monte-Carlo simulations for the stochastic analysis of funding problems. The results of Monte-Carlo simulations may materially depend on a number of technicalities. Most importantly, simulation based estimates are illustrations rather than conclusive quantitative demonstrations.

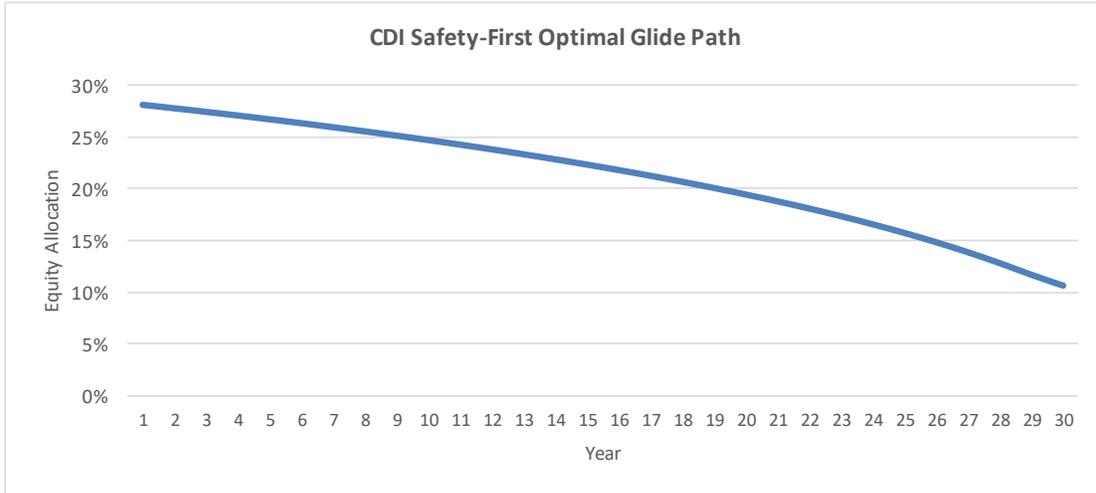
Glide Path 2.0 employs simulation-free analytical tools, i.e. the risks and uncertainties of funding problems are estimated without simulations. The results of simulation-free stochastic analysis are transparent and replicable. Simulation-free calculations are fast, efficient, and do not require an extensive computational infrastructure.

Ultimately, any quantitative glide path selection framework culminates in identifying the glide paths that optimize certain objective functions. In Glide Path 1.0, this identification is performed via a heuristic procedure known as “eyeballing.” *Pre-selected glide paths are inputs.* In Glide Path 2.0, optimal glide paths are generated by an optimizer. *Optimal glide paths are outputs.*

How would optimal glide paths for the funding problem discussed in this paper look in Glide Path 2.0? The optimal glide path under the “will-do” glide path assumption would look like the success rate maximizing glide path in *Exhibit 1*.

Under the “expected-to-do” glide path assumption, the Glide Path 2.0 toolbox would contain several optimization techniques. *Exhibit 7* presents the optimal glide path at 4% spending that delivers the highest flat success rate of 73% (assuming annual reevaluations).

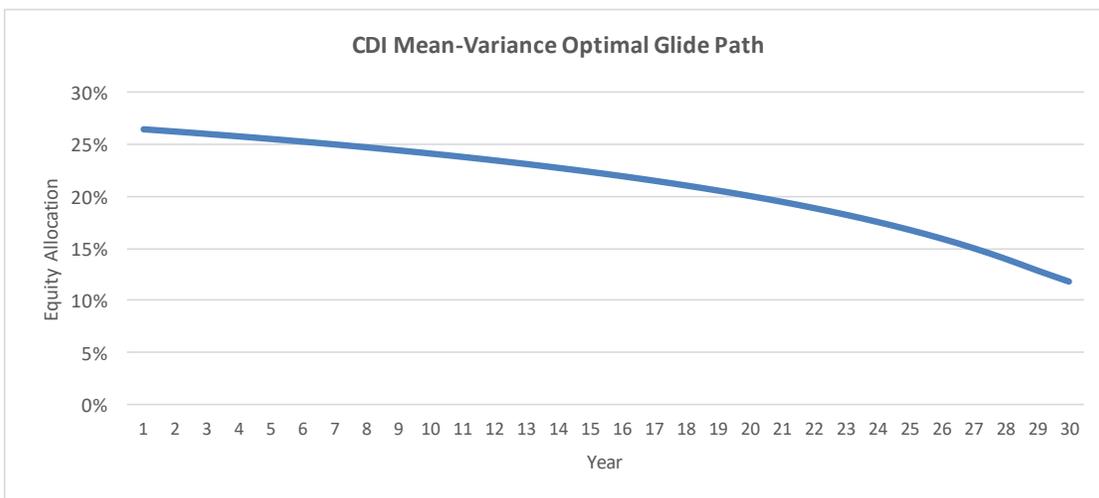
**Exhibit 7**



*Exhibit 8* presents the optimal glide path at 4% spending that delivers the highest flat risk aversion factor of 0.49 (assuming annual reevaluations).

The optimal glide paths presented in *Exhibits 7* and *8* are Nash equilibrium glide paths generated via the process of backward induction. Further technical details are outside of the scope of this paper.

**Exhibit 8**



**Conclusion**

By no means is this author suggesting that Pfau-Kitces (2014) is an exercise in futility. Pfau-Kitces (2014) is a great paper. Among other good aspects, this paper highlights the significance of investment objectives, capital market assumptions, and spending levels. Rising equity glide paths may be optimal under certain conditions.

Yet, the vast majority of target fund managers appear unimpressed. This author hypothesizes that the main reason for this lack of interest is that Pfau-Kitces (2014), despite its great qualities, is still little more than a collection of observations. Target date fund managers do not need more observations. They have plenty of their own.

In the absence of a paradigm-shifting upgrade from Glide Path 1.0 to 2.0, raging debates and confusion are likely to continue. Many investors that rely on conventional glide paths are likely to experience sub-optimal outcomes. The heuristics of Glide Path 1.0 may not deliver the efficient solutions investors need.

Dear reader, is it possible that the continued utilization of a deeply flawed framework may ultimately prove to be an exercise in futility?

**APPENDIX: Harold Evensky’s Capital Market Assumptions**

**Real Returns/Risk**

	Arithmetic Mean (%)	Standard Deviation (%)
Stocks	5.50	20.70
Bonds	1.75	6.50

**Correlation Matrix**

	Stocks	Bonds
Stocks	1	0.3
Bonds	0.3	1

*Source: Pfau-Kitces (2014)*

The success rate is estimated by means of calculating the first three moments of the terminal asset distribution and utilizing a matching theoretical distribution for the three moments.

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