LDI vs. CDI:

Bitter Foes or Best Friends?

Dimitry Mindlin, ASA, MAAA, Ph.D.
President

CDI Advisors LLC
dmindlin@cdiadvisors.com

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"I'm offering you my services, free of charge.
What do you say?"

Lieutenant Colonel Frank Slade, "Scent of a Woman"[1]

The term "Liability Driven Investing" (LDI) should be well-known in the retirement industry. After countless conferences, marketing presentations and publications, the term is a common buzzword. Most major asset management firms have LDI departments that employ "LDI strategists." Overall, the term LDI needs no introduction, and its meaning should be clear.

Today, it is perfectly reasonable to ask a plan sponsor "Are you using LDI?" and expect a simple "yes" or "no" answer rather than the oblivious "What the heck is LDI?" In a recent poll, 110 plan sponsors were asked whether they employed LDI. The result: the level of utilization of LDI is relatively high – 50% of the polled plan sponsors (think they) are currently using LDI.[2]

However, the poll contains some troubling information for the proponents of LDI. While the level of utilization of LDI is a respectable 50%, it is down from 54% a year ago. The meaning of the term LDI is still ambiguous enough to compel the pollsters to inquire the plan sponsors about their understanding of the term. Disturbingly, the most popular definition of LDI was different from the previous year – for the third year in a row! Furthermore, public plans – the last bastion of the DB industry – appear uninterested in LDI en masse.

For those practitioners (including this author) who believe that the "asset-only" approach to asset allocation is hopelessly inadequate, these developments are not terribly encouraging. The results of this poll, however, are not unexpected – they are just a reflection of fundamental problems with the presentation of LDI. It appears that the conventional version of LDI has limited appeal and requires structural renovations.

Clearly, the origins of LDI are sensible – investing of a retirement plan's assets should be "driven" by the plan's financial commitments. The true value of LDI as a framework for asset allocation decisions manifests itself in the investment solutions LDI supports. When it comes to funding financial commitments, *some LDI-supported financial products can be beneficial to some plans*. Yet, the proponents of LDI have offered neither a coherent justification for LDI solutions nor a clear definition of LDI as an economic concept. In particular, existing definitions of LDI are exceedingly vague, ranging from "a line of investment products to control the interest rate risk" to something virtually undistinguishable from the traditional ALM.

Most importantly, the proponents of LDI have failed to establish a direct relationship between LDI and the best interests of the stakeholders of retirement programs – the minimization of risk and cost of funding retirement benefits. Regrettably, the proponents of LDI have skipped over certain vital aspects in the development of a comprehensive approach to asset allocation LDI is proclaimed to be. Somewhere between LDI's origins and its practical recommendations, certain shortcuts have been taken; certain corners have been cut; certain unnecessary impractical assumptions have been made.

The purpose of this paper is to identify and correct these deficiencies. The paper presents a generalized version of LDI and expands its origins to create a solid foundation. The paper also presents powerful and flexible analytical tools that allow creating a transparent cost-risk management framework for retirement plans. As a result, retirement plans' decision makers would be able to identify investment solutions that serve the best interests of the stakeholders of retirement plans.

In a nutshell, the biggest problem with LDI is in the *L* part of the term – "Liability." LDI has major problems because the term "liability" is not properly defined in the context of asset allocation. Quite possibly, this term cannot be properly defined as an indisputably superior "one-size-fits-all" measurement in this context. The term "liability" has a multitude of meanings and tends to create a lot of confusion. Most importantly, it is unclear whether the term "liability" has anything to do with the objectives of retirement investing simply because LDI defines the term *before* it defines investment objectives.

In contrast, there is little ambiguity in the definition of the stream of benefit payments to plan participants.[3] This stream is called "*pension commitment*" (or just "commitment" for brevity) in this paper. There is no doubt that the commitment should play a significant role in the development of optimal policy portfolios. LDI, however, takes a major extra step – it jumps from the commitment to its deterministic measurement ("liability") with little concern for the consequences of the jump.

So, here are the key questions this paper poses.

What should "drive" investing of retirement assets – the stream of payments (the commitment) or its deterministic accounting measurement ("liability)? Should retirement plans utilize ***Commitment*** *Driven Investing (CDI) or* ***Liability*** *Driven Investing (LDI)?*

One of the main messages of this paper is the utilization of "liability" measurements of the commitment is unsubstantiated and likely to lead to inferior investment solutions. This paper makes the case for CDI and presents examples of CDI in action. However, the nuts and bolts of the CDI framework are outside of the scope of this paper (see Appendix I for a brief introduction to CDI; see Mindlin [2009a] and Mindlin [2009c] for more details).

LDI jumps from the "commitment" to its deterministic measurement ("liability") with remarkable ease.[4] The CDI framework unequivocally rejects this "easy" jump as unfounded and counterproductive. While LDI asserts that there is the single dominant measurement of the commitment – the "liability," – CDI maintains that the multitude of challenges retirement plans face require a multitude of measurements. Conceptually, LDI and CDI are at odds.

At the same time, as demonstrated in this paper, *CDI provides better support for LDI solutions than LDI*. The cost-risk minimization – the core of the CDI framework – offers superior rationale for portfolio selection in general and LDI solutions in particular than LDI's asset-"liability" matching. So, the question is: Are LDI and CDI bitter foes or best friends?

The Trouble with LDI

"If you make a mistake and get all tangled up,
you just tango on."

LDI is a relatively recent addition to financial terminology – the term was introduced in the early 2000s. A significant deterioration of the accounting measurements of most pension plans triggered a call to invest retirement assets in a way that would mimic accounting liabilities. Since some accounting liabilities have bond-like characteristics, some economists, actuaries and asset managers markedly intensified advocating bond-like investments for DB plans.

The logic behind the call was very simple. If the (accounting, "economic," "marked-to-market," etc.) "liability" behaves like a bond and asset-"liability" matching is desirable, which asset behaves like a bond? The answer is obvious – it is bonds and bond-like instruments.

The ostensible robustness and simplicity of this logic apparently was so attractive, that there began a large-scale marketing campaign to promote bond-like investments for DB plans. Right around that time, the term LDI was born.

However, several glaring flaws in this logic have always been evident. Some accounting "liabilities" do have some bond-like characteristics; some other "liabilities" have none; and no accounting "liability" behaves exactly like a portfolio of tradable bonds. Well, say the proponents of LDI, that is because these liabilities are not "marked-to-market." But there is no market for retirement benefits – those benefits are non-tradable and non-transferable (not yet, at least). Well, say the proponents of LDI, here is a scientific approach ("FE") that justifies the requirement of pricing retirement commitments like tradable bonds. But the FE approach is deeply flawed, possibly beyond repair (see Appendix III for more details). Well, say the proponents of LDI, most economists support it. But we do not resolve theoretical debates via democratic means – the support of "most" scientists does not necessarily makes a scientific theory true.[5] Sometimes it takes just one person to proclaim that the emperor has no clothes, and the debate ends right there.[6] Well, say the proponents of LDI to the financially unenlightened folks unimpressed with FE, you people go ahead and educate yourselves.

This is a concise summary of the wide-ranging debate that has been raging in economic and actuarial circles as well as some governing bodies. The debate continues to this day and shows no signs of abating. A shaky economic foundation and a raging debate do not set up a favorable environment for the acceptance of LDI products.

The trouble with LDI can be traced to several logical problems in its foundation. These problems include, but are not limited, to the following.

- The most consequential problem is LDI's lack of concern about the best interest of the stakeholders of retirement plans. Is the objective of matching assets and accounting "liabilities" the primary objective of a retirement plan? Would an LDI-generated optimal portfolio maximize the safety of benefits and/or minimize the cost of funding these benefits? Few, if any, publications present the investment objectives of LDI. Discussions of accounting and/or actuarial concepts as well as cash flow pricing notwithstanding, it is unclear what exactly the LDI framework is designed to achieve.

- The mistreatment the term "liability" receives in various publications regarding LDI is a major obstacle for a broader acceptance of LDI. In many cases, the definition of the term is either vague, or meaningless, or outsourced to accounting standard-setting bodies, or a combination of the above. Some reputable publications take too much liberty in their terminology and utilize the same term – "liability" – for *both* pension benefit payments and their present values.[7] This problem creates a lot of confusion and causes other problems in LDI. More examples of this problem in a popular textbook are presented in Appendix II.

- The claim that there exists a measurement of the commitment ("liability") that is superior to any other measurement and, therefore, must "drive" retirement investing defies common sense. Virtually any object has a multitude of measurements, and such a multifaceted object as a financial commitment is no exception. The best measurement *always* depends on the purpose of the measurement – the purpose comes first, the right measurement for the purpose is selected next. In LDI, this "order of operations" is reversed – a measurement is given first, purpose is given later (if ever). This issue is discussed in more detail later in the paper.

Few, if any, proponents of LDI have attempted to tackle these problems. LDI has gone little beyond its conventional motto "liability-is-a-bond-so-assets-must-be-in-bonds." It remains to be seen if the conventional version of LDI is capable of becoming a comprehensive and coherent approach to asset allocation. Count this author as an unabashed skeptic.

As of now, LDI proponents "tango on" on a very shaky foundation.

CDI vs. LDI: Objectives and Measurements

"*How's that for cornball?*"

The purpose of this section is to stress the importance of the following "chicken/egg" question. What comes first – defining investment objectives or measurements of the commitment?

Let us talk about investment objectives of a retirement plan. The plan's primary objective is to fund its financial commitment, and this commitment should "drive" investing of retirement assets. The investment objectives in the "commitment driven" environment are "driven" by the best interests of the plan's stakeholders and their risk tolerance. For example, plan participants may want to utilize the policy portfolio that maximizes the probability that the existing assets and a given level of contributions are sufficient to fund the commitment. Another example, taxpayers/shareholders may want to utilize the policy portfolio that minimizes the contributions required to fund the commitment within a given risk budget. These optimization problems require risk measurements of the commitment to achieve the investment objectives.

On the other hand, what would be the "driving" direction in the "liability driven" environment? One cannot even talk *a priori* about investment objectives in LDI because it is necessary to define the "best" measurement of the commitment ("liability") first. The next step would be to assume that it is desirable to match the assets and the "liability" and find the policy portfolio that behaves like the "liability" or something close to it.

One of the key differences between CDI and LDI is this "order of operations," where the "operations" are setting the investment objectives and defining the right measurement of the commitment. In CDI, the investment objectives – the cost/risk minimization – are specified first, and the right measurement is subsequently defined according to the objectives. In LDI, this order of operations is reversed – the "right" measurement ("liability") is defined first, and the investment objective – asset-"liability" matching – is defined next.

Thus, CDI and its " order of operations" lead to the policy portfolios that minimize the riskiness and cost of funding the commitment. In contrast, LDI and its "order of operations" lead to vaguely defined asset-"liability" matching. Cost/risk minimization and asset-"liability" matching are fundamentally different endeavors that are likely to lead to different optimal policy portfolios. As any elementary school student knows, the order of operations matters.

Dear reader, which order of operations makes the most sense to you?

CDI Framework: Controlling Interest Rate Risk

"*You've gotten so wrapped up in the sugar business,*
you've forgotten the taste of real honey!"

The goal of this section is to demonstrate how CDI's principles of cost/risk minimization lead to the selection of efficient portfolios when the interest rate risk is a key factor.

Think of a frozen retirement plan that wishes to purchase a group annuity contract for all accrued benefits in ten years.[8] For simplicity, let us assume that there are no benefit payments in the first ten years. The plan is assumed to have perfectly known future benefit payments, no future accruals, and the interest rate risk plays a major role in this funding problem. As a result, the example is essentially played on the home field of LDI. The choice of this example was also driven by the desire to make the results easy to understand and replicate.[9]

The plan's actuary has estimated the benefit stream to commence in ten years and determined that if today's interest rates remained the same, the price of this contract would be $100 in ten years. We assume that the plan's current asset value is $50, and the sponsor wishes to make a onetime additional contribution at the present in order to fund the commitment.

As we all know, interest rates fluctuate. As interest rates change, so does the expected price of the annuity contract. In year k, there is adjustment factor V_k due to the interest rate change in this year. Therefore, the plan's financial commitment is to make one payment in the amount of $\$100 \cdot V_1 \cdot \ldots \cdot V_{10}$ in ten years. This payment is equal to the original estimate $100 multiplied by the compounded adjustment factor $V_1 \cdot \ldots \cdot V_{10}$. We assume that the interest rate adjustments V_k are highly correlated with the price volatility of long bonds.

Furthermore, we assume that the assets are rebalanced to the same portfolio in all years from 1 to 10; this assumption will be relaxed later in this section. In this section, for simplicity, we consider the following three asset classes called "Equities," "Aggregate Bonds," and "LDI Solution":

1. Equities: benchmarked to broad indexes of the U.S. stocks (50%) and non-U.S. stocks (50%).
2. Aggregate Bonds: benchmarked to a broad aggregate index of fixed income products.
3. LDI Solution: a combination of LDI products that behaves like a bond portfolio whose price volatility is highly correlated with the price volatility of the annuity the plan intends to buy.

Capital market assumptions for all assets under consideration are presented in Appendix IV. Note that Aggregate Bonds and LDI Solution have the same geometric return; Aggregate Bonds are "shorter" and, therefore, are less "risky" in the "asset-only" space; LDI Solution has a higher correlation with the volatility of the commitment than Aggregate Bonds (0.9 vs. 0.4).

We also assume that the sponsor can purchase the desired annuity at the present as a deferred annuity at the price of $74.41 (=100/1.03^{10}, i.e. $100 discounted at 3%). Therefore, the risk-free asset requires the additional contribution of $24.41 (=$74.41 – $50.00). Now let us see whether risk taking can create value for the plan.

In this example, risk is defined as the *shortfall event* – the accumulated asset value in ten years is insufficient to purchase the desired annuity. There are several important measurements of the shortfall event – e.g. shortfall probability, shortfall size, shortfall volatility. For simplicity, the shortfall probability is designated as the primary risk measurement in this example.[10] It should be noted, however, that other measurements of the shortfall event are important as well.[11]

Let us define investment objectives for this funding problem. Generally, the sponsor wishes to minimize both the additional contribution and risk. However, this general sentiment is too imprecise to provide a solid basis for efficient portfolio selection. Lower contributions imply higher risk, and lower risk implies higher contributions, so contributions and risk are inseparable. The objectives to minimize contribution and risk need more accurate specifications. We recognize that the sponsor's objective is dual:

1. *To minimize contribution given risk*: for every value of shortfall probability, a portfolio that minimizes the additional contribution is generated.
2. *To minimize risk given contribution*: for every value of additional contribution, a portfolio that minimizes the shortfall probability is generated.

One of the key results of CDI is these objectives generate the same set of optimal portfolios. This set of portfolios is called *cost-risk efficient frontier*.[12]

Let us introduce the main technical tool that allows us to translate the funding problem into the standard language of probability theory. If C is an additional contribution, $r_1, \ldots, r_{10}$ are portfolio returns in years 1 to 10, and $R_k = 1 + r_k$ are return factors, then the shortfall probability is calculated as follows:

$$P = \Pr\left((50 + C)R_1 \ldots R_{10} < 100V_1 \ldots V_{10}\right) = \Pr\left(\frac{100V_1 \ldots V_{10}}{R_1 \ldots R_{10}} > 50 + C\right) \qquad (1)$$

Defining $RA = \frac{100V_1 \ldots V_{10}}{R_1 \ldots R_{10}}$, we get from (1):

$$P = \Pr(RA > 50 + C) \qquad (2)$$

Random variable *RA* (where *RA* stands for *Required Assets*) is the stochastic present value of payment $\$100 \cdot V_1 \ldots V_{10}$ funded by the portfolios that generate returns $r_1, \ldots, r_{10}$. For a given shortfall probability *P*, equation (2) shows that **50 + *C* is the (100% – *P*)th percentile of *RA***.[13]

In technical terms, the portfolio that "minimizes risk" given contribution *C* is the one for which 50 + *C* is the highest possible percentile of *RA*. Indeed, if 50 + *C* is the highest possible percentile of *RA*, then from (2) we have that 100% – *P* is the highest, so *P* is the lowest. Similarly, the portfolio that "minimizes contribution" given shortfall probability *P* is the one for which 50 + *C* is the lowest (100% – *P*)th percentile of *RA*.

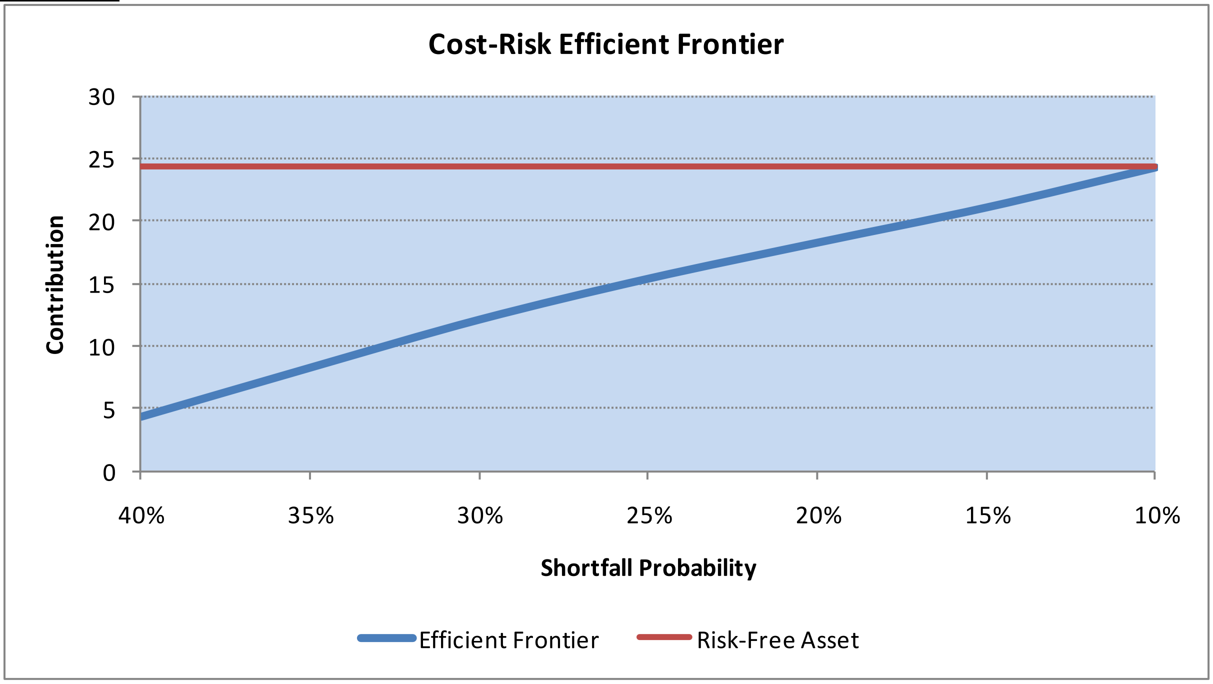
Now we are ready to specify the right present value measurement of the commitment. It is clear now that if the investor wishes to have shortfall probability *P*, then the most useful "present value" is the (100% – *P*)th percentile of *RA*, which is not a "present value" in a conventional sense, but a present value nonetheless – it is a value at the present. Clearly, this "present value" is instrumental in generating cost/risk efficient portfolios.

Exhibit 1 presents the cost-risk efficient frontier and compares the cost of risky portfolios to the cost of the risk-free asset. The cost-risk efficient frontier demonstrates the trade-off between risk taking and cost of funding the commitment. For example, for a reasonably risk averse investor (the shortfall probability of 20% or less), the lowest additional contribution is \$18.26, which is substantially lower (25%) than the risk-free contribution of \$24.41. *Exhibit 1* also shows that for a very risk averse investor (the shortfall probability of 10% or less), *the risk-free asset is the lowest cost investment solution*.[14]

Let us compare the cost-risk and mean-variance efficient portfolios. *Exhibit 2* compares a cost-risk efficient portfolio with two mean-variance efficient portfolios - the one that has the same expected return and the one that has the same "asset-only" risk.

The allocation of the CDI efficient portfolio for the shortfall probability of 20% is 34% Equities, 0% Aggregate Bonds and 66% LDI Solution (see *Exhibit 2*, Portfolio A). This portfolio has the arithmetic return of 6.29% and standard deviation of return of 9.13%. In contrast, the allocation of the mean-variance efficient portfolio that has the same arithmetic return 6.29% is 40% Equities, 60% Aggregate Bonds and 0% LDI Solution (see *Exhibit 2*, Portfolio B).

Exhibit 1



The allocation of the mean-variance efficient portfolio that has the same standard deviation of return 9.13% is 54% Equities, 44% Aggregate Bonds and 2% LDI Solution (see *Exhibit 2*, Portfolio C). Clearly, the mean-variance optimizer favors Aggregate Bonds over LDI Solution.

Now, let us compare the ability of portfolios A, B and C to fund the commitment. While portfolio A requires the additional contribution of $18.26, portfolio B requires $23.77 (30% increase) and portfolio C requires $22.69 (24% increase). Hence, as far as the funding problem is concerned, *portfolios B and C are significantly more "costly" than portfolio A*.

If the contribution is $18.26 for all three portfolios, then the shortfall probabilities are 27% and 25% for portfolios B and C correspondingly compared to 20% for portfolio A. As far as the funding problem is concerned, *portfolios B and C are significantly more "risky" than portfolio A*.

In this example, the concept of "risk" is illuminated from different angles. Having the same expected return, portfolio A is substantially "riskier" than portfolio B in the "asset-only" space (standard deviation of return of 9.13% vs. 7.35%), but substantially "safer" in terms of the funding problem (shortfall probability 20% vs. 27%). Thus, the "asset-only" concept of risk may not be appropriate for the funding problem. The primary reason for this conceptual unsuitability is the "asset-only" risk "knows" nothing about the interest rate risk, which is a major factor in the funding problem.

Exhibit 2

	CDI Efficient Portfolio A	Mean-Var Efficient Return Matching Portfolio B	Mean-Var Efficient Risk Matching Portfolio C
Global Equities	34%	40%	54%
Aggregate Bonds	0	60	44
LDI Solution	66	0	2
Total	100%	100%	100%
Arithmetic Return	**6.29%**	**6.29%**	6.88%
Geometric Return	5.90	6.03	6.50
St Dev of Return	**9.13**	7.35	**9.13**
Additional Contribution for Shortfall Probability 20%	$18.26	$23.77	$22.69
% of CDI Optimal Additional Contribution	100%	130%	124%
Shortfall Probability for CDI Efficient Contribution	20%	27%	25%
Risk-Free Additional Contribution	$24.41	$24.41	$24.41
% of Risk-Free Additional Contribution	75%	97%	93%

This example also demonstrates that the focus on investment return can be counterproductive, as higher expected return does not necessarily mean lower cost of funding. Indeed, portfolio C has much higher expected return than portfolio A (6.88% vs. 6.29%) and the same "asset-only" risk (9.13%). Yet, the cost of funding for portfolio A is lower than for portfolio C ($18.26 vs. $22.69) given shortfall probability of 20%. The primary reason for this outcome is portfolio A provides much better hedge for the volatility of the commitment.

The justification for the utilization of LDI Solution in this example stands in sharp contrast with the rationale for LDI products offered in a conventional version of LDI. The concept of accounting present value ("liability") has not been mentioned in the development of efficient solutions for this funding problem – it has been simply unnecessary. Nor has there been any discussion of either pension accounting, or asset-"liability" matching, or FE. The selection of

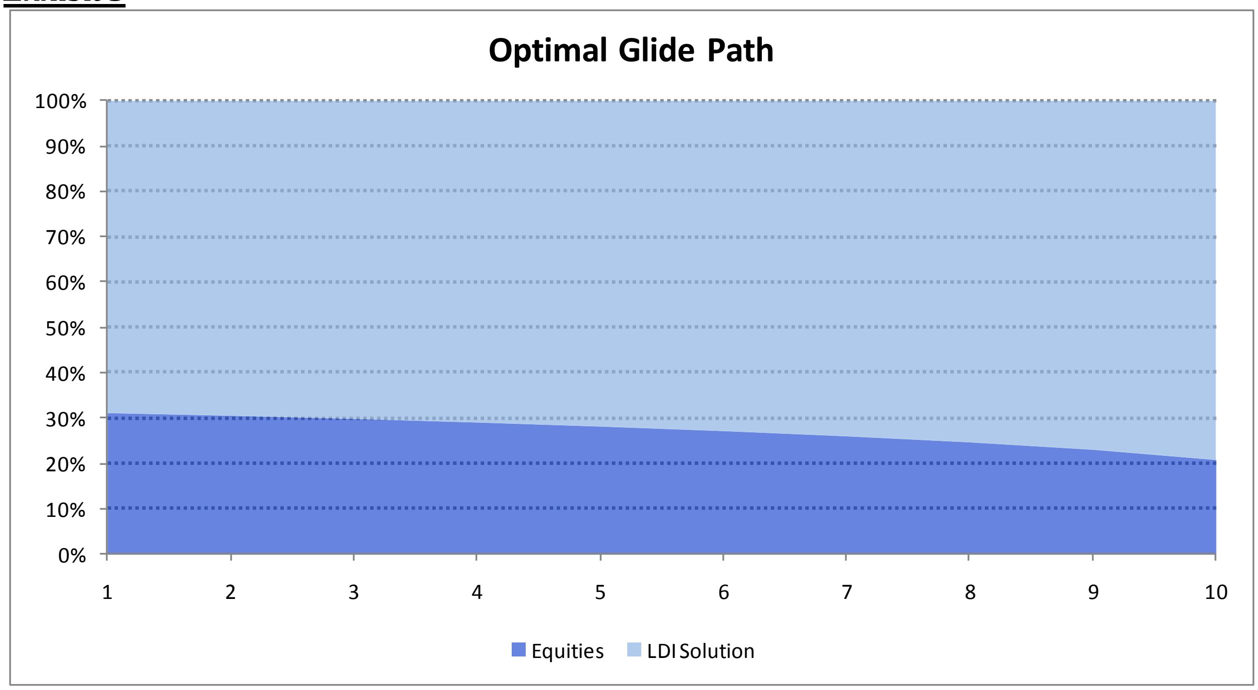
LDI Solution is based on the easy-to-grasp fact that LDI Solution reduces both the riskiness and cost of funding the commitment.

The example presented in this section viewed from a different angle contains a very important message for DC plan investors. Efficient portfolios for an annuity buyer can be very inefficient for an investor that has no intention of buying an annuity. Most DC plan investors do not currently buy annuities, and it is unclear when this situation will change, if ever. Yet, some publications on the subject of asset allocation for DC plans routinely assume that investors will purchase annuities in retirement. It should be emphasized that *the " annuity purchase" assumption matters a lot* – it has a major impact on optimal policy portfolios.

As promised, let us eliminate the assumption that the assets are rebalanced to the same portfolio in all years. Without this assumption, we are at liberty to consider the evolution of the investor's risk tolerance and design optimal multi-period strategy (a.k.a. *glide path*). For any series of shortfall probability values (*risk path*), CDI generates an optimal Nash equilibrium glide path.

Exhibit 3 contains the glide path associated with the risk path that starts with 79.7% and increases 2% every year to 97.7%. This risk path is chosen because it requires the same additional contribution $18.26 as a stationary "one- portfolio" glide path presented in *Exhibit 2*. Note that all portfolio in the optimal glide path have lower equity allocation than the stationary glide path. The optimal glide path's starting portfolio contains 31% of equities as opposed to 34% in the stationary glide path. The equity allocation in the optimal glide path ultimately decreases to 20.8%. Obviously, we have just scratched the surface as related to optimal glide path design (see Mindlin [2010b].for more details).

Exhibit 3



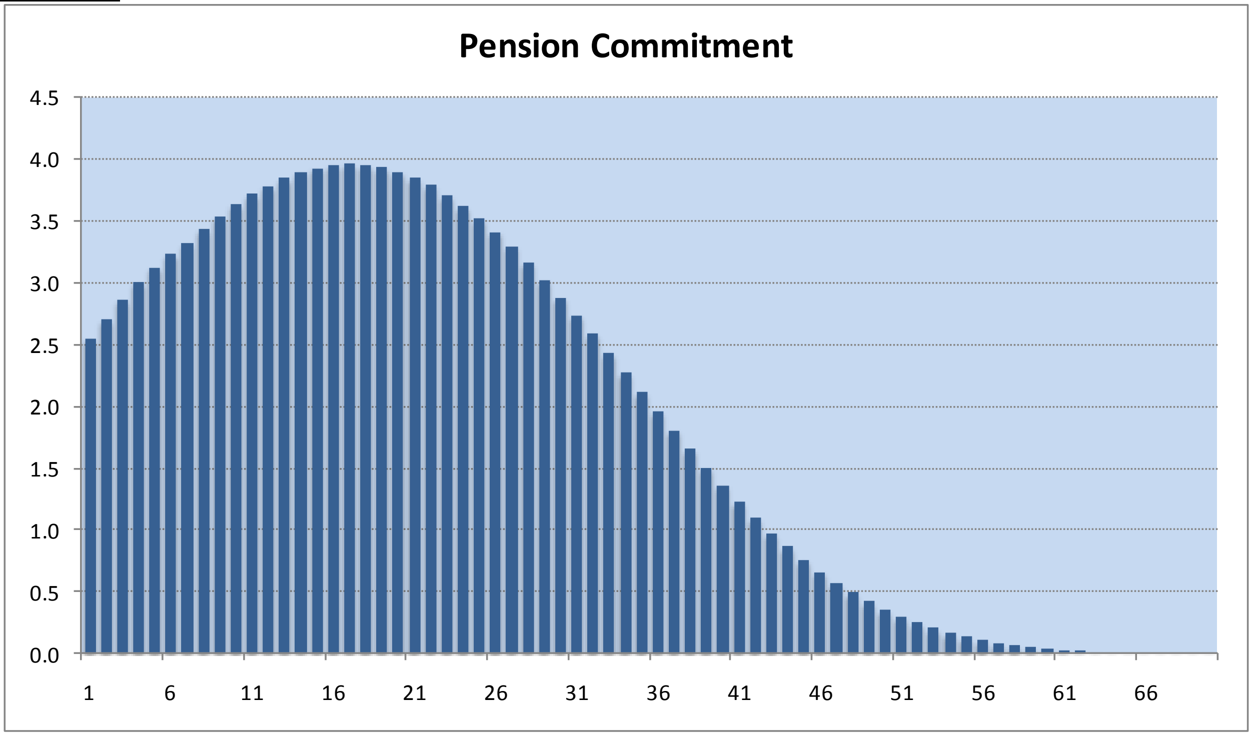
CDI Framework: Matching Bond Portfolios

"I'm just getting warmed up!"

The previous section demonstrated how LDI solutions for controlling interest rate risk naturally appear in the CDI framework. This section presents the initial steps in the analysis of another popular category of LDI products – matching bond portfolios – that naturally appear in the CDI framework as well.

Think of a frozen retirement plan that has predictable benefit payments and no future accruals. The plan's pension commitment is shown in *Exhibit 4*. We assume that the market value of the plan's assets is $50, and the present value of the commitment discounted at fixed rate of 6% is also $50.[15] Similar to the example in the previous section, we assume the sponsor wishes to make a onetime additional contribution at the present in order to fund the commitment.

Exhibit 4



To simplify this example even further, the assets are rebalanced to the same portfolio in all years. We assume that there are just two asset classes: equities and aggregate bonds described in the previous section. In this example, the focus is not on the selection of efficient portfolios – all portfolios are efficient since there are two asset classes only. The intention here is to separate the effect of efficient portfolio selection and concentrate on the selection of matching bond portfolios. This example demonstrates how principles of cost-risk minimization help determine whether a "buy-and-hold" portfolio is in the best interests of the plan's stakeholders.

Similar to the previous section, for simplicity, the shortfall probability is designated as the primary risk measurement in this example.[16] Once again, it should be noted that other measurements of the shortfall event are important as well.

As discussed in the previous section, the sponsor wishes to minimize contribution given risk and minimize risk given contribution. Specifically, for every value of shortfall probability, we generate a portfolio that minimizes the additional contribution; for every value of additional contribution, we generate a portfolio that minimizes the shortfall probability. These objectives generate the same set of optimal portfolios – *the cost-risk efficient frontier*.

Let us define the stochastic present values associated with this funding problem. If $r_1, \ldots, r_n$ are portfolio returns in years 1 to *n*, $R_k = 1 + r_k$ are return factors, and $P_1, \ldots, P_n$ are the benefit payments, then stochastic present value *RA* of the plan's pension commitment is defined as follows.

$$RA = \sum_{k=1}^{n} \frac{P_k}{R_1 \ldots R_k} \tag{3}$$

If *C* is an additional contribution, then the shortfall probability is calculated similar to (2):

$$P = \Pr(RA > 50 + C) \tag{4}$$

To produce the cost-risk efficient frontier, we find the portfolio that generates is the lowest (100% – *P*)th percentile of *RA* for each shortfall probability value *P*.

The presence of a "buy-and-hold" bond portfolio creates a new optimization problem and requires a couple of adjustments to the original optimization problem. If $B_1, \ldots, B_n$ are the payments of a bond portfolio, then stochastic present value *RA* of the plan's pension commitment offset by the bond payments is defined as follows.

$$RA = \sum_{k=1}^{n} \frac{P_k - B_k}{R_1 \ldots R_k} \tag{5}$$

If *L* is the price of the bond portfolio today, then the plan's asset value is equal to 50 – *L* + *C*. The shortfall probability is calculated similar to (2) and (4):

$$P = \Pr(RA > 50 - L + C) \tag{6}$$

We consider three bond portfolio – the ones that match the pension commitment for the first 5, 10 and 15 years. Today's prices of these portfolios are \$12.84, \$25.77 and \$37.72.[17] *Exhibit 5* contains the resulting four cost-risk efficient frontiers (namely, no matching, 5, 10 and 15 year matching).

Exhibit 5 demonstrates that matching "buy-and-hold" portfolios are more "costly" when the level of risk aversion is low. As the level of risk aversion increases, the frontiers get closer and, at some point, cross each other. After this point, matching portfolios provide lower cost of funding.

Exhibit 5

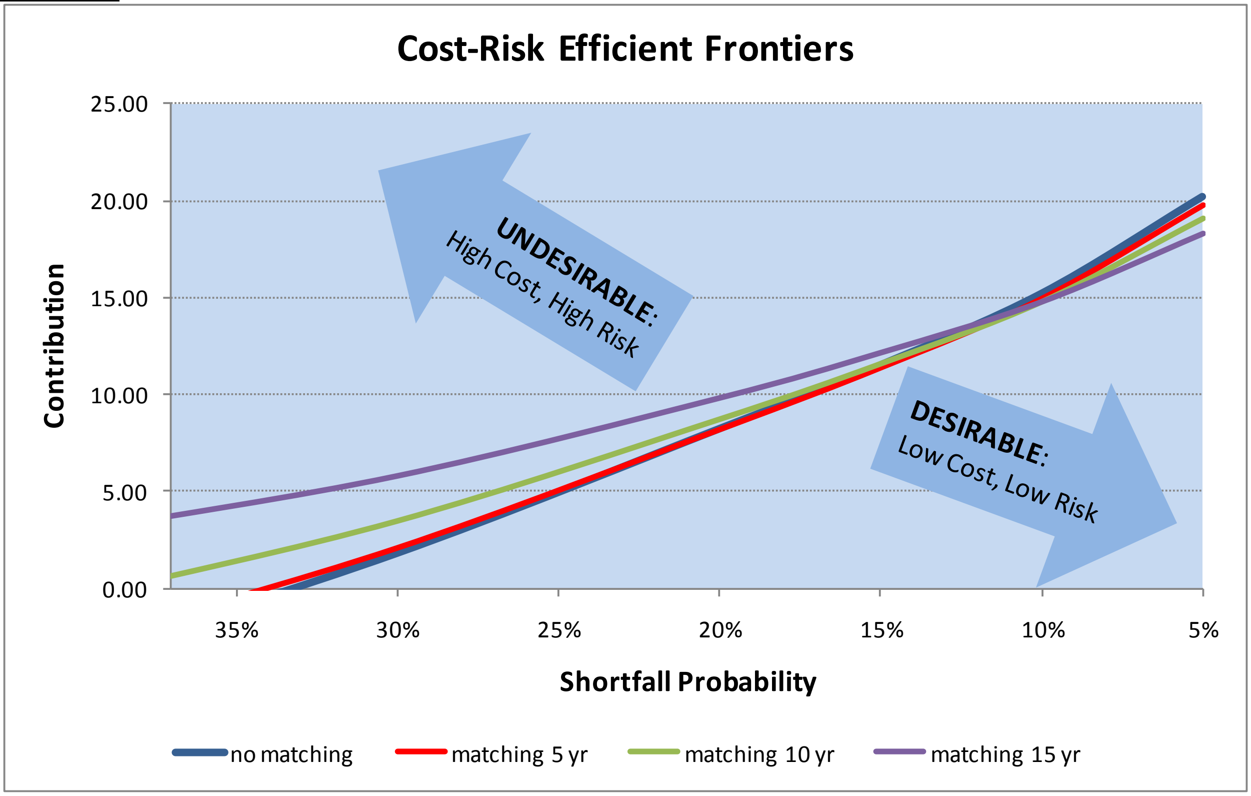


Exhibit 6 reinforces this message. If the desired shortfall probability is 30%, then the "no matching" strategy delivers the lowest required contribution of $1.78. For the shortfall probability of 20%, the "no matching" strategy still delivers the lowest "cost" of $8.21, but the "cost" with 5 year matching is very close ($8.22). The strategy with 5 year matching delivers the lowest cost of $11.41 for the shortfall probability of 15%; the one with 10 year matching delivers the lowest cost of $14.78 for the shortfall probability of 10%; the one with 15 year matching delivers the lowest cost of $18.34 for the shortfall probability of 5%.

It should be emphasized that this section presents just initial steps in the analysis of "buy-and-hold" assets. Further steps should include the analysis of shortfall size and volatility of various strategies, but these steps are outside of the scope of this paper.

Once again, the cost/risk minimization – the core of the CDI framework – provides easy-to-grasp decision making tools for the selection of efficient portfolios. Once again, the core of the LDI framework – accounting and other "liabilities" as well as asset-"liability" matching – are simply unnecessary.

Exhibit 6

Lowest Additional Contributions						
	Shortfall Prob 40%	Shortfall Prob 30%	Shortfall Prob 20%	Shortfall Prob 15%	Shortfall Prob 10%	Shortfall Prob 5%
No Matching	$0.00	**$1.78**	**$8.21**	$11.50	$15.20	$20.21
Matching 5 yr	0.00	2.10	8.22	**11.41**	14.98	19.80
Matching 10 yr	0.00	3.46	8.71	11.58	**14.78**	19.08
Matching 15 yr	2.91	5.81	9.84	12.18	14.81	**18.34**

CDI vs. LDI for DC Plans

“*But there is nothing like the sight of an amputated spirit.*”

Originally, LDI was introduced for defined benefit (DB) plans only. These days, however, the term "LDI" is increasingly being used in discussions regarding post-retirement asset allocation decisions for defined contribution (DC) plan participants. In the DC plan environment, the flaws of LDI are especially glaring.

The case for LDI in the post-retirement phase usually starts with a critique of the "asset-only" approach. Next, there is a discussion of post-retirement needs, the cash flows associated with these needs, and the role these cash flows should play in the development of optimal policy portfolios. All these matters are perfectly sensible, outside of the fact that modern portfolio theory – a quintessential "asset-only" approach – is usually utilized in the pre-retirement phase and promptly denounced in the post-retirement phase.

As soon as the term "liability" appears, the conventional LDI pitch unfolds rapidly. One may start by stating "Retirement expenses form the retiree’s retirement liability."[18] Then, after declaring that the investor's portfolio should be considered in its entirety – all investor's assets and liabilities (which are also assets, although held short) should be included, – these assets and "liabilities" can be managed simultaneously using traditional methodologies.

Dear reader, did you see the LDI "jump" in this logic? Did you notice how the stream of post-retirement spending got "present valued" and became a bond-like return-generating asset? Once again, a non-tradable cash flow is assumed "similar to debt" and, therefore, priced like a tradable bond with no justification at all.

This logic in general and the LDI "jump" in particular are especially out of place in the DC plan environment. This approach is in conflict with the realities and the spirit of DC plans. Individuals do not have "liabilities" in a conventional sense. For instance, if a DC plan investor has endeavored to make 70% of the participant's pre-retirement income available in retirement and failed to do so, no one is "liable" for anything.

The presence of LDI in the post-retirement phase, but not in the pre-retirement phase, creates an internally inconsistent model that fails to deal with the best interests of the plan's stakeholders directly. Also, the presence of LDI in the post-retirement phase necessarily assumes that the retiree has purchased an annuity, which may or may not be a realistic assumption. As a result, LDI is likely to lead to sub-optimal investment solutions. An efficient policy portfolio for a purchaser of an annuity is likely to be inefficient otherwise, as was demonstrated in a prior section.

DC plan participants may not have conventional liabilities, but they certainly make financial commitments. A DC plan participant may make a commitment to fund 70% of the participant's pre-retirement income in retirement. In order to achieve this goal, the participant may make a commitment to save a fixed or increasing percentage of the participant's income. The plan sponsor may make a commitment to make matching contributions to the participant's retirement account. All these commitments are somewhat similar – they are contingent cash flows of uncertain timing and magnitude.

All these commitments are not legally binding and can be easily modified or discontinued if necessary. The term "liability" is inappropriate to any of these commitments, therefore the term "LDI" and the essence of the LDI framework – matching assets and bond-like "liability" – are inapplicable. In contrast, CDI can seamlessly integrate these and other commitments the participant may have (e.g. medical and educational expenses, charitable contributions, bequest, etc.) in a unified internally consistent model that takes into account the best interests of the plan's stakeholders directly.

CDI contains a powerful and flexible multi-period portfolio optimizer that generates Nash equilibrium investment strategies. This multi-period methodology is possible precisely because CDI, as a cash flow based framework, does not "present value" financial commitments before the investment objectives are stated.

The application of the principles of CDI has already led to important developments in the asset allocation area. First, CDI provides the solution for the decades-old Samuelson-Merton time diversification problem – it explains why an optimal glide path should evolve, with or without "human capital," mean-reversion, or expenses (see Mindlin [2009b] for more details). Second, the CDI framework provides the foundation for arguably the first systematic approach to optimal glide path design – from basic principles to practical conclusions (see Mindlin [2010b]). This author is optimistic that CDI will eventually become the mainstream approach to funding financial commitments.

Conclusion

> "*You're in no position to disagree with me, boy.*
> *I got a loaded .45 here. You got pimples.*"

There are major challenges in the asset allocation area for institutional and individual investors. The need for innovative solutions is obvious. The asset allocation theory for DC plans is in its infancy; the asset allocation theory for DB plans needs major renovations. Undoubtedly, the development of new methodologies will require a great deal of creative thinking.

The advent of LDI was a recognition that the existing asset allocation methodologies were inadequate in the early 2000s. LDI was an attempt to take a step in the right direction. Unfortunately, LDI in its current form has certain fundamental deficiencies that undermine LDI's prospects of becoming a universally accepted asset allocation methodology.

Do I believe that the presence of the term "LDI" in the industry will diminish anytime soon? No. Do I believe that CDI will start competing with LDI anytime soon? No. LDI is an established concept, however imperfect, with extensive institutional support. In contrast, CDI is a new concept that is unlikely to become as prominent as LDI. Not anytime soon.

Yet, I am cautiously optimistic that the proponents of LDI would realize that the principles of CDI significantly improve the conventional LDI presentation. This paper outlines possible improvements that would enhance the appeal of LDI and the acceptance of its solutions. As long as the improved framework provides better service to retirement plan participants and sponsors, it makes little difference whether the framework is called LDI, CDI, or something else.

To recap, here are a couple of key messages. A retirement plan's commitment – a series of payments to the plan participants – should be incorporated in the development of optimal policy portfolios directly. The plan should recognize the best interests of its stakeholders, establish investment objectives and risk "budget," identify the right risk measurements of the commitment consistent with the objectives, and develop optimal policy portfolios using these measurements.

An investment committee that endeavors *to maximize the safety of retirement benefits and minimize the cost of funding these benefits* should produce much better results than a committee that spends most of its time chasing the hottest asset classes with the highest expected returns and worrying about the volatility of accounting statements. In pursuit of the right approach, the committee members should "keep their eyes on the ball" and be open to exploring new ideas. While there is no broadly accepted asset allocation methodology for cost/risk minimization just now, CDI should be considered as a candidate. Or so I hope.

The proponents of LDI should recognize that the most important "liability" the committee members may want to keep in mind is the fiduciary one. The committee members may look favorably upon well-substantiated investment solutions that are beneficial to the plan's objectives, thank you very much.

APPENDIX I

Commitment Driven Investing: a Foreword

Financial commitments play a special role in investing. Investors do not invest in a vacuum – they contribute to their investment programs and take a multitude of risks mainly to fund their financial commitments. A commitment – a future cash flow the investors strive to fund – is the primary reason a particular investment program exists in the first place. The objective of funding the commitment is the driving force behind the asset allocation and contribution strategies as well as the guiding light for risk taking.

Over the last several decades, institutional and individual investors have managed their investment programs primarily via the risk/return analysis of their portfolios consistent with modern portfolio theory (MPT). Due to recent experience, a growing number of practitioners are beginning to question the ability of MPT to produce meaningful recommendations to investors. After all, MPT is a quintessential *portfolio* theory – it "knows" nothing about a particular investor's specific financial commitments as well as certain essential aspects of the investor's circumstances (e.g. existing asset value, contribution rate, compensation, demographic characteristics). Regardless of these commitments, MPT performs one-period optimization of *future* asset values assuming that the asset value is known at the present.

For an investor with financial commitments to fund, the challenge is exactly the opposite. The future values – the commitments – are given. The challenges are at the present – to develop optimal asset allocation and contribution policies as related to the funding problem. There is a need for a new framework with a multi-period portfolio optimization methodology that incorporates the commitments and delivers optimal solutions for these challenges.

Financial commitments, their magnitude, timing and volatilities are at the core of this framework. To highlight the dominant role of financial commitments in this framework, the framework is called *Commitment Driven Investing* (CDI).[19] It is assumed that the investor's primary objective is to fund the commitment – to ensure that the money is readily available every time a payment is due.

The investors' objectives are one of the cornerstones of CDI. The CDI framework is based on the following common-sense assumptions regarding investors' objectives:

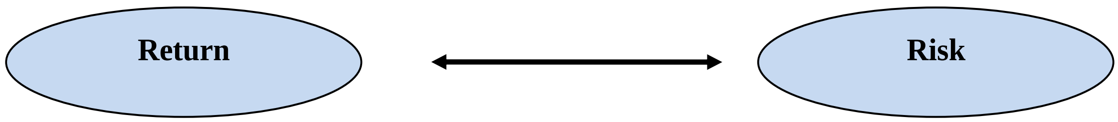
- to maximize the sustainable commitment;
- to minimize the riskiness of funding the commitment;
- to minimize the cost of funding the commitment.

Thus, investors objectives in the CDI framework have three major components: commitment, cost, and risk.

In contrast, MPT concentrates on the analysis of two major components of the portfolio optimization problem: return and risk. The picture is two-dimensional (see *Exhibit 7*).

Exhibit 7

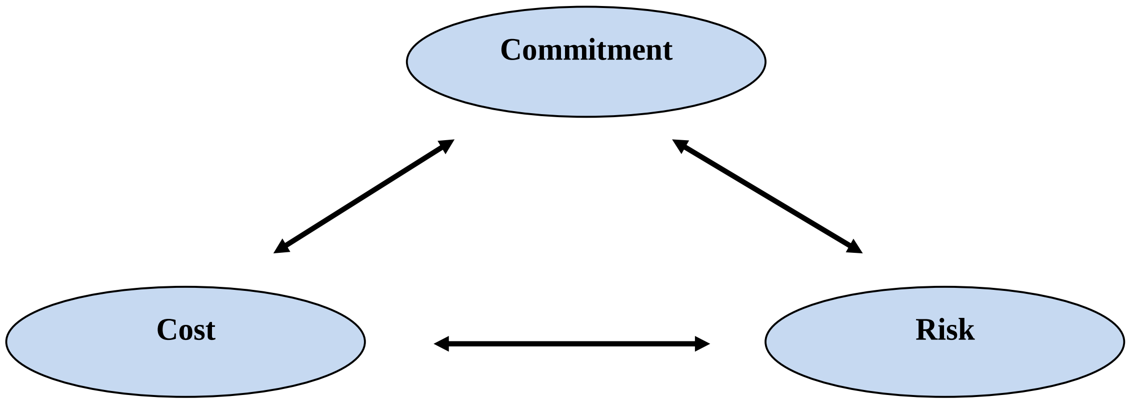
The Risk-Return Line



The presence of a financial commitment changes this picture dramatically. The commitment itself represents a new component. The return considerations become a subset of more encompassing cost considerations. The concept of risk obviously remains, but becomes much more comprehensive. As a result, the funding problem involves three components: commitment, cost, and risk, which form *the funding triangle*. The picture is three-dimensional (see *Exhibit 8*).

Exhibit 8

The Funding Triangle



All three components of the funding triangle and the relationships between them are indispensable for finding optimal solutions to the funding problem. In particular, there is no risk management without cost analysis; there is no cost management without risk analysis. The commitment must be feasible given the resources the investor is willing to contribute and the risk the investor is willing to take. The cost must be reasonable given the commitment and the risk budget. The risk must be tolerable given the commitment and the cost structure.

Clearly, there are fundamental relationships between all three components of the funding triangle. The stakeholders of investment programs have to manage all three components at the same time, even though investors may not have total control over these components.

One of the most important parts of the funding problem – asset allocation – may appear to be hidden in the funding triangle, but only seemingly so. Asset allocation plays a major role in the management of all components of the triangle. Asset allocation is one of the most important means in the optimization of the funding triangle. Asset allocation is in fact one of the main aspects of CDI.

It is informative to look at the optimization objectives in the two-dimensional "Risk/Return" framework. There, the objective of portfolio optimization is either "minimize risk given return" or "maximize return given risk." In other words, one component of the line is optimized given the other component. It can be shown that both objectives lead to the same set of optimal policies (*the mean-variance efficient frontier*).

The situation in the funding triangle "Commitment/Risk/Cost" is similar, although more complex. There are several different objectives for the optimization of the funding triangle. Similar to the two-dimensional case, these objectives are formulated according to the principle "given two components, optimize the third".

The objective of maximizing the commitment given cost and risk is applicable to a DC plan participant who wishes to maximize her standard of living in retirement. The objective of minimizing risk given cost and commitment is applicable to DB plan participants who wish to maximize the safety of their benefits. The objective of minimizing cost given risk and commitment is applicable to taxpayers/shareholders who wish to minimize the cost of running a DB plan.

One of the key properties of the funding triangle is, under certain conditions, optimization objectives lead to the same set of optimal investment strategies. For example, given a DB plan's pension commitment, the objectives of "minimizing cost given risk" and "minimizing risk given cost" lead to the same optimal policy portfolios (*the cost-risk efficient frontier*). Therefore, as far as asset allocation is concerned, there is no conflict between the best interests of plan participants (safety of benefits) and taxpayers/shareholders (low cost of funding).

For more details about Commitment Driven Investing, see Mindlin [2009a] and Mindlin [2009c].

APPENDIX II

The Term "Liability" in a CFA Textbook

"*I always knew what the right path was.*
Without exception, I knew.
But I never took it. You know why?
It was too damn hard."

The tendency to utilize the term "liability" in an undisciplined manner is widespread and largely overlooked. Numerous publications give different definitions of "liabilities," but even more publications give no definition at all. This tendency is as regrettable as it is out of sync with common academic standards.

This appendix contains several examples of the misuse of the term “liability.” The appendix is written as an illustration of the abovementioned unfortunate tendency and largely for the reader’s entertainment.[20]

CFA Institute has published a textbook “Managing Investment Portfolios” (third edition, 2007) that presents the basics of asset-only and asset-liability methodologies in its Chapter 5 “Asset Allocation” (called the Chapter in this section). Naturally, the Chapter utilizes the concept of “liability” for the development of optimal portfolios. The comments in this appendix are limited to the use of the term “liability” in the Chapter; this appendix does not attempt to review other aspects of the Chapter.

The Chapter presents the principles of asset-liability management and discusses the objectives of asset allocation and the role of the policy portfolio for investors with financial commitments to fund. When it comes to the concept of "liability," unfortunately, the Chapter is very confusing with respect to the following basic questions: *Why should the concept of "liability" be used in the development of optimal portfolios? What is the “liability”? Is the “liability” a cash flow or a present value of a cash flow?*

In the spirit of full disclosure, I should mention that I communicated my concerns regarding the misuse of the term “liability” to the authors of the Chapter when a preliminary version of the Chapter was posted. In his response, one of the co-authors indicated that he understood the concerns. However, he responded that the choice of somewhat imprecise terminology was justified because the Chapter was so “introductory.” Perfecting the terminology would make the text too hard to grasp for beginners; it would be much harder to write as well. My attempt to convince the co-author that a clear presentation of basic concepts would be especially important for beginners was unsuccessful.

The textbook sends clear messages regarding the concept of "liabilities" prior to the Chapter. In the first definition of the “pension surplus,” the need to “present value” those “future liabilities” is perfectly clear (page 66).

“*The pension surplus equals pension plan assets at market value minus **the present value of pension plan liabilities***.”[21]

The presence of “the present value” in this definition is absolutely necessary. “Pension liabilities” are future payments made at different points in time, and these payments must be “present valued” in order to be subtracted from today’s assets.

A statement on page 69 reiterates the need for the “present value” in this definition.

“*For a DB plan, one key ALM concept is the pension surplus, defined as pension assets at market value minus **the present value of pension liabilities***.”

At the beginning, the Chapter is clear about the meaning of the term "liabilities." On page 236, “future liabilities” are obviously a cash flow:

"*… insurers, defined benefit (DB) pension plans, and certain other institutional investors face **streams of significant future liabilities**. Controlling the risk related to **funding future liabilities** is a key investment objective for such investor."*

The emphasis on funding continues on the same page. These "future liabilities" are instrumental in the development of optimal asset allocation:

"*In the context of determining a strategic asset allocation, the asset–liability management (ALM) approach involves explicitly modeling liabilities and adopting the optimal asset allocation in relationship to **funding liabilities**."*

A statement on page 237 reiterates once again that "pension liabilities" require "present valuing."

"*DB pension plan may want to maximize the future risk-adjusted value of pension surplus (the value of pension assets minus **the present value of pension liabilities**)*."

The meaning of the term "funding" is further clarified on page 286, where "a liability" is a future payment:

"*... **funding a liability** means being able to pay the liability **when it comes due***."

The emphasis on funding as a multi-period endeavor in these statements is of paramount importance. According to these definitions, the investor's job is to make sure there is enough money to make all payments when they come due in the future. In this context, "future liabilities" and the "commitment" as defined in this paper are synonymous.

The whole thing starts unraveling on page 237. The "present value" mysteriously disappears in the following statement:

"*ALM strategies run from those which seek to minimize risk with respect to net worth or surplus (**assets minus liabilities**) to those which deliberately bear surplus risk in exchange for higher expected surplus*."

Readers beware, those "liabilities" cannot be subtracted from the assets – it is against the basic principle of the time value of money. As defined in the textbook, "liabilities" are benefit payments made at different points in time, while the asset value is at the present. To be able to subtract "liabilities" from "assets," one must calculate a present value of these "liabilities."

Unfortunately, as presented in the Chapter, the term "liabilities" represents entirely different objects. This is not just a minor slip of a tongue – the Chapter contains multiple occurrences of the term "liabilities" utilized to represent either a cash flow or a present value. Even worse, in some cases, the reader has to guess the meaning of the term. Below are just a few examples of these multiple meanings with additional comments.

- "Liabilities" is a present value (page 237): "*ALM strategies run from those which seek to minimize risk with respect to net worth or surplus (assets minus liabilities) to those which deliberately bear surplus risk in exchange for higher expected surplus, analogous to the trade-off of absolute risk for absolute mean return in an asset-only approach.*" The objectives of "funding future liabilities" and "minimizing the surplus risk" are fundamentally different. They are based on different assumptions and may lead to different optimal policy portfolios.
- "Liabilities" is a cash flow (page 237): "*A cash flow matching approach structures investments in bonds to match (offset) future **liabilities** or quasi-liabilities. When feasible, cash flow matching minimizes risk relative to funding **liabilities**.*"
- "Liabilities" is either a present value or a cash flow (page 238): "*In general, the ALM approach tends to be favored when … the market value of liabilities or quasi-liabilities are interest rate sensitive*." This statement is controversial for several reasons. First, pension benefits are non-tradable and non-transferable, so "the market value of liabilities" in a conventional sense does not exist. Second, if the term "liabilities" in this sentence means a cash flow, then their benefit payments are not directly interest rate sensitive. In this case, this statement implies that "the ALM approach" should not "be favored" by retirement plans – a silly proposition. Third, if "the liabilities" in this sentence mean a present value, then the sensitivity to interest rates must be justified somehow, which the Chapter does not do.
- "Liabilities" either a present value or a cash flow (page 245): "*Shortfall risk in relation to liabilities is a key focus of ALM approaches to asset allocation.*" What a great point! The ability of a particular asset value to fund a stream of future payments is surely one of the main challenges in the area of asset allocation. But this point is immediately followed by the following incredible statement: "*An asset-only approach can also easily incorporate shortfall risk in a variety of ways.*" Really? Dear authors, do you honestly believe that "an asset-only approach" can calculate, for example, the shortfall risk – the probability that the commitment will not be funded? Apparently, the term "shortfall risk" in the statement regarding "a key focus of ALM approaches" is actually the "surplus shortfall risk." In this case, the statement is both incorrect and in conflict with the abovementioned statement on page 236: "*Controlling the risk related to **funding future liabilities** is a key investment objective for such investor.*" Readers, beware: "funding future liabilities" and surplus risk management are fundamentally different endeavors.
- "Liabilities" is a present value (page 255): "Net worth" is again defined as "***assets minus liabilities***." We are back in the "present value" mode.
- "Liabilities" is a cash flow (page 286): "*In many cases, however, an asset portfolio is meant **to fund a specified liability schedule** (funding a liability means being able to pay the liability **when it comes due**).*" Again, this is a very important clarification.
- "Liabilities" is a present value (page 287): "*A single variable that summarizes the interaction of assets and liabilities is net worth (**the difference between the market value of assets and liabilities**), also called surplus.*" We are back in the "present value" mode once again.
- "Liabilities" is a cash flow (page 288): "*The funding ratio, calculated by dividing the value of pension assets by **the present value of pension liabilities**, measures the relative size of pension assets compared with pension liabilities.*" The "present value" reappears.

As the Chapter continues, it gets even worse. It is troubling enough that a stream of "liabilities" gets "present valued" and becomes just a "liability" for no particular reason. Now this present value of a non-tradable cash flow conveniently turns into an asset (held short) for no particular reason as well. The implied assumptions are a) this benefit stream can be replicated using a portfolio of tradable bonds; b) the only legitimate present value of the commitment is the price of this hypothetical bond portfolio. The Chapter makes no attempt to justify these assumptions.

Here are a few examples of this convenient transformation.

- Page 290: "*Assume that the* ***return on the pension liability*** *tracks the return on U.S. government long-term bonds.*" Suddenly, the pension liability *is assumed* to be a return-generating asset.
- Page 291: "*The pension liability behaves as a long-term bond, by assumption.*" Interestingly, a similar statement is presented as a "fact" in Bader-Gold [2003].[22] Much to the authors' credit, they do not go that far and reiterate that the similarity of "the pension liability" and "a long-term bond" *is assumed*.
- Page 308: "*Pension liabilities behave similarly to bonds as concerns interest rate sensitivity*." Finally, the similarity of "pension liabilities" and bonds is stated unequivocally as a fact. The proponents of LDI have reached the Promised Land.

What a mess!

Yet, it would be unwise to put the blame for the problems in the Chapter on its authors. The purpose of a textbook is to reflect the state of affairs in the field rather than revamp the field. It is likely that the authors understood the problems with their use of the term "liabilities," but resolving those problems was neither easy nor the authors' goal. The Chapter is essentially a survey of the existing publications on the subject – the Chapter simply communicates what the authors see out there. Let us not blame the messenger for the trouble with the message.

APPENDIX III

The FE Approach

"*How come you always get that wrong?*
Because it's not important for me to get it right."

To bridge the gap between the commitment and its "liability" measurement(s), the proponents of LDI have advanced a presumably scientific approach (called "FE approach") that attempts to justify the superiority of a particular "liability" measurement over all other measurements of the commitment. As a result, the FE approach is in fact the heart and soul of LDI. The FE approach has been extensively debated among economists, actuaries, and many others in recent years.[23] Therefore, this appendix contains just a brief discussion of the approach.

One of the most unfortunate aspects of the debate regarding the FE approach is the lack of a coherent presentation of the approach. This author is aware of just one serious attempt to present the foundation of FE in a scientifically rigorous manner (Bader-Gold [2003]), and this attempt ultimately failed to achieve its objectives.[24] Since then, the proponents of the approach have offered little more than generalized pronouncements in the vein of "every-economist-knows-this" and calls for more education.[25] The proponents of the approach appear to have little interest in building a solid foundation for their views. There is no doubt that if the proponents of the FE approach tried to "get it right" – to present the FE approach systematically from basic principles to practical conclusions, – they would promptly discover that this is a major (and, possibly, insurmountable) challenge.

The FE approach is based on the following two heroic assumptions.

1. For any pension commitment, there exists a matching portfolio of tradable bonds.
2. The price of this portfolio is the only valid present value of the commitment.

The first assumption is plainly unreasonable for ongoing plans. It is a bit more realistic for the plans that wish to wind up in the foreseeable future (to close, freeze, or terminate). Consequently, to make this assumption at least remotely sensible, the majority of LDI proponents advocate the utilization of termination-like "liability" that is based on existing service and compensation and no future benefit accruals.

The basis for the second assumption is unclear. Some proponents of FE insist that this assumption *is* based on the Law of One Price; some other proponents of FE assert that this assumption *is not* based on the Law of One Price. This author has been anxiously awaiting the final clarification of this matter, but exceedingly skeptical that it is coming anytime soon.

The second assumption is essentially an asset allocation preference, not a theoretical concept. Some proponents of the (presumably scientific) FE approach declare that the approach necessitates all-bonds allocation for pension plans.[26] As a theoretical concept, the requirement to use the (hypothetical) risk-free asset as a benchmark is unsubstantiated. Jointly, these assumptions essentially promote the view that solvency concerns must dominate retirement plan management – another debatable proposition.

The next step would be to require some kind of asset-"liability" matching, so this "liability" would essentially "drive" the investment of retirement assets into the LDI environment. The proponents of LDI provide no reasons for the belief that such asset-"liability" matching is in the best interests of retirement plans' stakeholders.

As we see, the desire to turn a commitment into a bond-like "liability" requires some additional considerations and debatable assumptions. In the absence of this desire, these assumptions are unnecessary. In particular, there is no need for these assumptions in the CDI framework. Interestingly, there is no need for these assumptions in the LDI framework either, at least as far as LDI solutions are concerned. As demonstrated in this paper, LDI solutions are perfectly justifiable the CDI framework dominated by cost-risk considerations rather than "liabilities."

LDI and FE seen from a different angle are particularly amusing. Based on some accounting and/or pricing considerations, it is *assumed* that the starting point of LDI and FE – the present value of the commitment ("liability") – is deterministic. The asset value required to fund the commitment (a.k.a. present value) is deterministic – known with certainty – only when the assets are invested in a matching bond portfolio and/or other fixed income assets. Any mismatch or uncertainty of return would result in a non-zero volatility of the present value, which is assumed to be zero. Thus, by virtue of using deterministic present values, the FE approach essentially *assumes* that the assets are invested in bonds.

Then, after some elaborate reasoning, the proponents of FE conclude that pension assets should be invested in bonds.[27] So, here is the gem of FE: *if we assume that the assets are invested in bonds, then the assets should be invested in bonds*. Dear reader, does this application of FE strike you as an important achievement? Do you appreciate the value of FE and the need for more education in this area? Did I mention that the FE approach is presumably scientific?

Overall, the FE approach has a dubious theoretical foundation, utilizes unreasonable assumptions, and, as a result, fails to provide a solid foundation for LDI. Similar to the problems with LDI discussed in prior sections, however, these issues do not appear to be on the radar screen of the proponents of LDI.

One may view the relationship between CDI and LDI as *CDI is LDI without FE*.

APPENDIX IV

Capital Market Assumptions

Expected Return/Risk

	Geometric Return	Arithmetic Return	Standard Deviation
Equities	7.70	8.81	15.68
Aggregate Bonds	4.50	4.62	5.00
LDI Solution	4.50	4.97	10.00
Commitment Volatility	0.00	0.49	10.00

Correlation Matrix

	Equities	Aggregate Bonds	LDI Solution	Commitment Volatility
Equities	1.00	0.16	0.16	0.10
Aggregate Bonds	0.16	1.00	0.60	0.40
LDI Solution	0.16	0.60	1.00	0.90
Commitment Volatility	0.10	0.40	0.90	1.00

All portfolios and volatilities are assumed to be lognormally distributed.

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Important Information

ENDNOTES

[1] The epigraphs to this and all other sections in this paper are quotations of Lieutenant Colonel Frank Slade, the character played by Al Pacino in "Scent of a Woman" (1992). For more details about the movie, see http://www.imdb.com/title/tt0105323/.

[2] The poll was conducted by SEI, a global provider of outsourced financial services and included pension executives from the Netherlands, United Kingdom and United States. The results were reported at http://www.seic.com/enUS/about/4521.htm and Plan Sponsor, December 2, 2010, LDI Adoption Steady, Definition "Fluid," http://www.plansponsor.com/LDI_adoption_steady_definition_fluid.aspx.

[3] In some cases, more than one commitment is associated with a retirement plan. Depending on the plan's circumstances, one may select the termination-like benefit stream (current service and compensation) or the going concern benefit stream (past and future service, future compensation). These technicalities, however, are outside of the scope of this paper.

[4] To justify the jump, LDI utilizes so-called "FE approach," which is discussed in Appendix III. The term "FE" comes from the claim made by some economists and actuaries that the FE approach takes its roots in financial economics. This author disputes this claim and distinguishes FE and financial economics.

[5] The following is a comment on this subject from Mindlin [2010a]: "*Science is not a democratic institution. Scientists do not resolve their disagreements by plebiscite, acclamation, voice vote, or any other democratic means. To a courteous scientific debate, scientists contribute books and scholarly articles, which gain recognition via the quality of their contents. In the presence of quality academic publications, any "consensus" declaration is needless. In the absence of quality academic publications, any "consensus" declaration is useless. Either way, the claim "every economist knows this" is an inconsequential line of reasoning as well as a clear sign of weakness of one's arguments.*"

[6] The following is another comment on this subject from Mindlin [2010a]: "*At some point, most scientists were still in agreement that the earth was the center of the universe, but this agreement did not last forever. A bit later, most scientists were still in agreement that the earth was flat. As evidenced by the declining membership in the Flat Earth Society, this agreement did not last forever as well. As far as scientific facts are concerned, the existence of such "agreements" is irrelevant. However, as far as the lives of some disagreeing dilettantes were concerned, those "agreements" were highly relevant at the time.*"

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[7] One of the most illuminating manifestations of this problem is the definition of "liabilities" in Ryan, Fabozzi [2002]: "… the *liabilities* are valued as the present value of future *liabilities* … ." This recursive definition of the term "liabilities" provides little information and may in fact lead to sub-optimal investment solutions.
[8] Alternatively, think of an individual investor that wishes to retire and buy an annuity in ten years.
[9] The phrase "easy to replicate" means the results of this section can be replicated using college-level math and basic Microsoft Excel programming.
[10] This is an example of the Safety-First version of the CDI framework. See Mindlin [2009a] for more details.
[11] Ironically, this author finds himself somewhat inconsistent in this section. While the author emphasizes the need for precise language when it comes to "liability," the author is not that vigilant when it comes to "risk." Risk is defined as an *event*, yet the authors talks about "risk minimization" throughout the paper. One can minimize risk *measurements* of an event, but not the event itself. Hence, it would be more accurate to talk about "risk measurement minimization" rather than "risk minimization." However, this language is routinely used in financial literature, and this issue would be an unwanted distraction from the main message of the paper. So, the author chooses to ignore this "inconsistency." Or, at least. in this paper.
[12] See Mindlin [2009a] for a formal proof of this statement.
[13] It is assumed that probability *P* is expressed in percentage terms. In this paper, it is also assumed that the distributions of portfolio returns and volatilities are continuous.
[14] More precisely, the "break-even" point is the shortfall probability of 9.76%.
[15] Let us not call this plan "fully funded," which is popular but highly misleading term.
[16] Again, this is an example of the Safety-First version of the CDI framework. See Mindlin [2009a] for more details.
[17] To calculate these prices, we assume a simplified yield curve: 3.5% for years 1 to 5, 3.75% for years 6 to 10, and 4.0% for years 10 to 15.
[18] What a beauty! See Idzorek [2008]. If we put "in retirement" at the end of this sentence, it would read "Retirement expenses form the retiree's retirement liability in retirement." Even better.
[19] For a basic introduction to the CDI framework, see Mindlin [2009e].
[20] I can easily imagine an incredulous reader saying something like this. "You call this stuff entertainment? Who do you think reads this hokum?" Fair enough, let me take back "for the *reader*'s entertainment." The appendix is written largely for the *author*'s entertainment.
[21] In this and all other quotations, emphasis is mine - DM.
[22] See Bader-Gold [2003], page 4. Speaking of Principle 4 (which is arguably the most important point of the paper), the authors assert that "This principle follows from the fact that a company's pension liabilities are similar to debt."
[23] See for example, McCrory, Bartel [2003], Blake, Khorsanee [2005], Mindlin [2005], Mindlin [2006], Mindlin [2007], Mindlin [2008a], Mindlin [2008b], Mindlin [2010a].
[24] See Mindlin [2005] for a comprehensive rebuttal of Bader-Gold [2003].
[25] SOA-AAA [2006] is another attempt to present certain elements of the FE approach. However, SOA-AAA [2006] is a collection of miscellaneous observations regarding corporate pension finance rather than a scientifically rigorous academic publication. Unlike Bader-Gold [2003] that called for a wholesale reinvention of pension actuarial science, SOA-AAA [2006] is "not a statement about how actuaries should practice nor how pension plans should be operated."
[26] For example, see Bader, Gold [2007].
[27] See Bader, Gold [2007].