



Why Glide Paths Evolve: “Expected-to-Do” Glide Paths

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ABSTRACT

This is the first article in a two-part series entitled “Why Glide Paths Evolve.” The series discusses one of the most fundamental questions of optimal glide path design – should optimal glide paths be evolving or stationary?

This article demonstrates that the investor’s glide path should generally evolve if the investor endeavors to achieve the best outcomes and expects to regularly reevaluate his portfolio makeup. The article presents a concise review of the pros and cons of the existing justifications for evolving glide paths. The article also presents a simple example that demonstrates that the principle “any sub-glide path of an optimal glide path should be optimal on its own” generally leads to evolving glide paths. We call the glide paths that follow this principle “expected-to-do” glide paths because they are based on rational expectations of the investor’s future portfolio selections. The second article in the series will present different conditions that also lead to evolving glide paths.

The term “glide path” takes its roots in aviation. On the surface, therefore, the title of this series may sound like an oxymoron. After all, an aircraft’s glide path signifies an orderly evolution of altitude from flying to landing. Glide paths evolve by definition.

For *investment portfolio* glide paths, however, the situation is entirely different. While millions of investors rely on portfolio glide paths, there is no consensus among glide path designers regarding portfolio selection and evolution. Even the most basic aspects of optimal glide path design ignite raging debates.

In particular, the fundamental question regarding the desirability of portfolio evolution – *should optimal glide paths be evolving or stationary?* – has been controversial for a long time. While most glide path designers agree that glide paths should evolve, there is no consensus regarding the reasons for glide path evolution and its directions. Despite numerous publications on the subject, many major issues remain unresolved.

The need for a solid foundation for glide path portfolio selection and evolution is well-understood in the industry. In particular, the managers of Target Date Funds (TDF) – a popular and rapidly growing type of investment products that utilize glide paths – conduct a lot of research in this area. The competition for the best glide path design framework is intense. A recent paper from *Russell Investments* gives a fair assessment of the current situation:

“The battle over intellectual authority over the management of TDF assets in retirement will have increasingly high stakes.”¹

One of the key aspects of this “intellectual authority” should be a clear understanding of the reasons for optimal glide path evolution. Without understanding of these reasons, the glide path evolution “free lunch” would be unavailable, thus preventing investors from achieving the best outcomes.

Such understanding, however, is hard to find in current publications on glide path design. Most publications steer clear of the subject of glide path evolution. The distinct minority of glide path designers that tackle this subject do so in a way that leaves a lot to be desired. A couple of popular attempts to justify evolving glide paths are discussed in the next section.

So, why should glide paths evolve? Before we answer this question, we should determine the proper form that an answer to this question should take. Our position is that a proper answer to this question should be in the form of a sensible glide path design framework. *The utility of such a framework fundamentally depends on the principles that serve as the framework's foundation.*

We hold that *Commitment Driven Investing* (CDI), a quantitative framework designed to generate optimal asset allocation, contribution, and payout strategies for investors with financial commitments to fund, represents such a framework. The essentials of CDI are presented in [Mindlin \[2014\]](#). A simplified example of the CDI framework is presented in this article.

This article demonstrates that the following principle of CDI naturally leads to evolving glide paths.

*Any sub-glide path of an optimal glide path should be optimal on its own.*²

This article is organized as follows. First, the article provides a brief overview of the popular justifications for evolving glide paths as well as their pros and cons. Second, the article presents a simple numerical example that illustrates the multi-period process of portfolio selection that results in evolving glide paths. The article concludes with a brief discussion of the state of affairs in the industry.

Evolving Glide Paths: Popular Justifications

Financial economists have endeavored to rationalize evolving glide paths for decades.³ Yet, the results of these efforts have largely failed to satisfy industry practitioners and academics alike and settle this issue.

There are two well-known justifications for evolving glide paths. The first (and the oldest) one is based on the perception that the young have more time to ride out the volatility of equity markets. Therefore, the young should invest mostly in stocks and shift to bonds as they get older. This justification is closely related to the controversial “time diversification” property of stocks (i.e. the riskiness of stocks diminishes with time). Many economists do not believe that this justification is adequate, even though some practitioners still occasionally promote it. See [Mindlin \[2009\]](#) for more details.

The second justification (a.k.a. the “human capital” theory) is based on the following assumptions:

1. There are two types of capital: financial and “human.” Human capital is defined as the present value of current and future income over the investor’s remaining time horizon.
2. The human capital is assumed to behave like a portfolio of conventional assets and “should be treated like any other asset class.”⁴
3. The investor’s capital in its entirety (i.e. financial and human capital) should form well-diversified portfolios throughout the life-cycle. If the human capital is assumed to behave like a bond portfolio, then the young should invest mostly in stocks and shift to bonds as the bond-like human capital diminishes over time.

Some of these assumptions are quite problematic. Human capital is the ability to earn current and future income; it is a cash flow, not a present value. The need to “price” human capital and the similarity of human capital to conventional assets are unsubstantiated. The objective of producing well-diversified portfolios may not be directly related to producing optimal outcomes. Most importantly, glide paths generated by the human capital theory may produce sub-optimal outcomes (see [Mindlin \[2015 B\]](#)).⁵

Yet, despite these and other problems, the human capital theory is currently the most popular approach to glide path design in the accumulation phase. The primary reason for this popularity is not hard to grasp – there is little else out there. One of the goals of this article is to change this perception. The next section makes an important step in this direction.

Evolving Glide Paths: An Example

The example in this section has a specific logical structure and “order of definitions” that are very important. The “order of definitions” is as follows:

1. *The investor’s problem.* The investor intends to invest \$1 for two years with no additional contributions. The investor intends to make two asset allocation decisions – at the present and at the beginning of the second year, so the investor’s glide path would have two portfolios. The investor wishes to utilize two asset classes – stocks and bonds. The capital market assumptions are presented in the appendix.

2. *Risk*. Risk is defined as *the shortfall event* – the value of investor’s assets is lower than a pre-determined (terminal) asset value at a pre-determined point in time.
3. *Risk measurements*. The *shortfall probability* is designated as the primary risk measurement.⁶ The investor believes that a 30% shortfall probability is appropriate for his purposes.
4. *Investment objectives*. The primary objective for portfolio selection is to *maximize the terminal asset value* that falls within the pre-determined risk “budget” (i.e. has a 30% shortfall probability).

Let us take a closer look at the intended asset allocation decisions – at the present (called decision *A*) and at the beginning of the second year (called decision *B*). Decision *B* involves just one portfolio selection. Since the primary objective involves the terminal asset value, decision *A* requires portfolio selections in years one and two. Therefore, decision *A* uses decision *B*, so the investor should make decision *B* first and make decision *A* next.

What the investor should rationally expect decision *B* to be? This is the key point in this example. The principle “any sub-glide path of an optimal glide path should be optimal on its own” implies that this decision should require a portfolio optimization procedure that is based on a risk tolerance assumption.⁷

For simplicity, let us assume that the investor intends to exercise *the same risk tolerance profile* (the definition of risk, the primary risk measurement, and the target shortfall probability) for both asset allocation decisions. The objective is to select a portfolio that maximizes the terminal asset value that has a 30% shortfall probability. Therefore, *the investor should select a portfolio that generates the highest 30th percentile of the terminal asset value*. Straightforward calculations demonstrate that this portfolio has 19.8% of stocks and 80.2% of bonds.⁸

As the next step, let us make decision *A* (i.e. select the optimal portfolio for the first year). Given the 19.8/80.2 portfolio in the second year, what portfolio in the first year maximizes the terminal asset value in two years that has a 30% shortfall probability? In other words, given the 19.8/80.2 portfolio in the second year, what portfolio in the first year generates the highest 30th percentile of the terminal asset value in two years? Equally straightforward calculations demonstrate that this portfolio has 26.7% of stocks and 73.3% of bonds.⁹

Let us recap these results. The optimal glide path was generated via the process of *backward induction*, i.e. the second year portfolio was selected first, and the first year portfolio was selected next. Under identical risk tolerance conditions and the assumption that risk tolerance is exercised throughout the investor's time horizon, the optimal glide path is evolving.

Here are several additional observations regarding this example.

- This example is deliberately simplified to highlight the key concepts as well as make it replicable and easy to understand. A thorough exposition of these issues requires a higher level of quantitative analysis than suitable for this article. See [Mindlin \[2009\]](#), [Mindlin \[2013\]](#), [Mindlin \[2014\]](#), and [Mindlin \[2015 B\]](#) for more details.
- Conventional glide paths currently utilized in the industry are based on the assumption that the investor will always dutifully follow the glide path designed today. We call such glide paths "*will-do*" glide paths. In contrast, the glide path in this example is based on rational expectations of the investor's future optimal portfolio selections. We call such glide paths "*expected-to-do*" glide paths. This example demonstrates that optimal "*expected-to-do*" glide paths are generally evolving. In subsequent articles, we demonstrate that optimal "*will-do*" glide paths are generally evolving as well. It should be noted that we believe that "*expected-to-do*" glide paths better reflect the realities of long-term investing.
- This example emphasizes the importance of the definition of risk in the context of an investment problem. Risks in the "asset-only" and "asset-commitment" spaces are fundamentally different. *Constant risk in the "asset-commitment" space may imply evolving risk in the "asset-only" space.*
- In this example, the assumption of constant risk tolerance was made for simplicity only. The investor is at liberty to choose different risk profiles in different time periods.
- In this example, the equity allocation in first year is greater than its second year counterpart. However, decreasing equity allocations are not a general rule. Optimal glide paths may have different shapes under different assumptions. See [Mindlin \[2015 B\]](#) for more details.
- Optimal glide paths evolve regardless of the presence of "human capital." The human capital assumptions discussed in the previous section are unnecessary.

The great late Paul Samuelson spent four decades looking for a sensible framework that justifies evolving glide paths. In honor of his efforts, we call this challenge the *Samuelson problem*.¹⁰ Thus, the CDI framework in the example discussed in this section represents a solution to the Samuelson problem.

Conclusion

As demonstrated in this article, certain principles of optimal glide path design lead to evolving glide paths. Other principles may lead to stationary glide paths (e.g. Samuelson [1969]). Therefore, the principles of glide path design frameworks should be the center of attention for investors in general and glide path designers in particular.

Anyone interested in reviewing these principles in current publications on the subject, however, would surely find it challenging. Few publications discuss the principles of their glide path design methodologies. Instead, many publications offer vaguely defined concepts, questionable observations, and debatable logic that may lead to problematic glide path design decisions.¹¹ Most importantly, these decisions may not be directly related to the objective of generating the best outcomes and, therefore may not be in the investors' best interests.

Obviously, this article has just scratched the surface. It is clear, however, that the conceptual foundation of glide path design and related issues require a major cleanup. Without it, millions of investors would likely utilize portfolios that generate sub-optimal outcomes.

Once again, why should glide paths evolve? To answer this question, one should present a sensible framework that is based on rational principles of investing. The answer to this question presented in this article is that the principle "*any sub-glide path of an optimal glide path should be optimal on its own*" generally leads to evolving glide paths. Glide paths designed according to this principle – "expected-to-do" glide paths – are generally evolving, with or without constant risk aversion or human capital.

The upcoming second article in the series will demonstrate that "will-do" glide paths are also generally evolving.

APPENDIX: Capital Market Assumptions**Return/Risk**

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Stocks	7.00	8.03	16.00
Bonds	4.00	4.12	5.00

Correlation Matrix

	Stocks	Bonds
Stocks	1	0.2
Bonds	0.2	1

Lognormal distributions are used to approximate portfolio returns.

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¹ See Knowles [2015].

² See [Mindlin \[2015 A\]](#) for more details about this principle.

³ The works of Paul Samuelson in this area are especially notable. For example, see Samuelson [1969], Samuelson [1991], and Samuelson [1992]. Also, see [Mindlin \[2013\]](#) for more details.

⁴ See Ibbotson [2007] for more details.

⁵ [Mindlin \[2015 B\]](#) also provides a more detailed examination of the justifications for evolving glide paths discussed in this section.

⁶ The designation of the shortfall probability as the primary risk measurement is for simplicity only. Other risk measurements (e.g. shortfall size and volatility) are important as well.

⁷ The principle “risk tolerance is exercised throughout the investor’s time horizon” would lead to the same conclusion.

⁸ The technical details of these calculations are available upon request (certain conditions apply).

⁹ See the previous endnote.

¹⁰ See [Mindlin \[2013\]](#) for more details.

¹¹ For example, see Daverman [2014].

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