Why Glide Paths Evolve: "Will-Do" Glide Paths

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ABSTRACT

This is the second article of a two-part series entitled "Why Glide Paths Evolve." The series discusses one of the most fundamental questions of optimal glide path design – should optimal glide paths be evolving or stationary?

The first article presented a framework that was based on the principle "any sub-glide path of an optimal glide path should be optimal on its own." We call the glide paths that follow this principle "expected-to-do" glide paths because they are based on rational expectations of the investor's future portfolio selections. The article demonstrated that "expected-to-do" glide paths are generally evolving.

In contrast, this article assumes that the investor will always dutifully follow the glide path designed today. We call such glide paths "will-do" glide paths. This article demonstrates that "will-do" glide paths are generally evolving as well.

This article continues the discussion of one of the most fundamental questions of optimal glide path design – should optimal glide paths be evolving or stationary? We started this discussion in Mindlin [2015 C], the first part of a two-part series entitled "Why Glide Paths Evolve."

Mindlin [2015 C] reviews of the pros and cons of the existing justifications for evolving glide paths and maintains that a proper justification for evolving glide paths should be in the form of a sensible framework in which optimal glide paths are generally evolving. The utility of such a framework fundamentally depends on the assumptions and principles that serve as the framework's foundation.

Mindlin [2015 C] considers two such assumptions. First, it is assumed that the investor wishes to achieve the best outcomes. This assumption is self-evident and needs no validation. Second, the investor expects to regularly reevaluate his portfolio makeup in the future. This seemingly innocuous assumption is of great consequence and plays a major role in the process of optimal glide path design.

In particular, the regular reevaluation assumption implies that the investor is expected to optimize remaining glide paths at all future reevaluation points. This observation leads to the principle "any sub-glide path of an optimal glide path should be optimal on its own." We call the glide paths that follow this principle "*expected-to-do*" glide paths. As demonstrated in Mindlin [2015 C], optimal "expected-to-do" glide paths are generally evolving.

"Expected-to-do" glide paths are based on rational expectations of the investor's future portfolio selections. In contrast, conventional glide paths currently utilized in the industry normally make no assumption regarding future portfolio selections. Instead, it is assumed that the investor will always dutifully follow the glide path designed today. We call such glide paths "*will-do*" glide paths. We believe that "expected-to-do" glide paths better reflect the realities of long-term investing. However, the current prevalence of "will-do" glide paths in the industry compels us to consider these glide paths and their properties.

In this article, therefore, we employ an optimal glide path design framework that is not based on rational expectations of future portfolio selections. Instead, the "will-do" glide paths generated in this framework are based on the mindset of Modern Portfolio Theory (MPT) applied to multi-period problems. The next section demonstrates that the glide paths generated in this framework are generally evolving.

Evolving "Will-Do" Glide Path: An Example

This section presents a simplified example of a glide path design problem. This example was selected primarily to present an easily replicable example that requires neither advanced math nor sophisticated modeling. More realistic problems will be discussed in future publications.

Let us consider an investor that has $2 and a commitment to make two payments of $1 one year and two years from now. Let us assume the investor has decided to invest in stock and bond index funds only. Loosely speaking, the investor's objective is to maximize the asset value after two years; we call this asset value "*terminal value*" in this paper. The terminal value is uncertain since the investment returns are uncertain. Therefore, suitable measurements of the terminal value are needed in order to properly define the optimization objective.

To do so, let us employ an approach similar to MPT. In MPT, one of the key investment objectives is to maximize the mean of the future value of $1 given the volatility of the future value. Let us employ a similar investment objective to our (two-period) problem. Specifically, let us define the optimization objective as *to maximize the mean of the terminal value given the volatility of the terminal value*.

In this paper, the measurements of terminal value are calculated using the simulation-free stochastic analysis methodology developed at *CDI Advisors LLC*. For the reader's convenience, the formulas for the mean and volatility of the terminal value are presented in Appendix I. The capital market assumptions utilized in this article are presented in Appendix II.

As in MPT, we use various volatilities of terminal value. Consequently, we need to quantify the aforementioned "given" volatilities of terminal value. To do so, let us consider ten "given" stationary glide paths – from 0% to 90% equity in 10% increments – and their terminal values. *Exhibit 1* presents the key measurements of these terminal values.

Exhibit 1

Given equity allocation										
Year 1	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%
Year 2	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%
Mean	0.127	0.139	0.152	0.164	0.177	0.190	0.202	0.215	0.228	0.241
Volatility	0.117	0.118	0.129	0.148	0.173	0.201	0.232	0.265	0.299	0.334

For each of the given stationary glide paths, we calculate the optimal glide path that has the highest mean of terminal value and the same terminal value volatility as the give glide path. *Exhibit 2* expands *Exhibit 1* and presents optimal glide paths and their measurements.

Exhibit 2

Given equity allocation										
Year 1	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%
Year 2	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%
Mean	0.127	0.139	0.152	0.164	0.177	0.190	0.202	0.215	0.228	0.241
Volatility	0.117	0.118	0.129	0.148	0.173	0.201	0.232	0.265	0.299	0.334

Optimal equity allocation										
Year 1	8%	9%	17%	25%	33%	41%	49%	58%	73%	87%
Year 2	9%	12%	27%	42%	58%	73%	88%	100%	100%	100%
Mean	0.137	0.140	0.153	0.166	0.179	0.192	0.205	0.218	0.231	0.242
Volatility	0.117	0.118	0.129	0.148	0.173	0.201	0.232	0.265	0.299	0.334

Exhibit 2 demonstrates that optimal glide paths are evolving. However, for this conclusion to be credible, we should answer the following important question. How significant are the mean terminal value increases generated by the optimal glide paths? For instance, the terminal value mean for the stationary 50% equity glide path is 0.190. Its counterpart generated by the glide path with equity allocations of 41% (year 1) and 73% (year 1) is 0.192 (with the same volatility). How significant is the mean terminal value increase from 0.190 to 0.192?

One way to answer this question is to reduce the expected return for the 41%/73% glide path by the amount necessary to reduce the terminal value mean from 0.192 to 0.190. This expected return reduction can be interpreted as an annual "surcharge" implied by the inefficiencies of the given glide path. *Exhibit 3* expands *Exhibit 2* and presents these "surcharges" (measured in basis points) for all glide paths. In particular, the stationary 50%/50% glide path implies an annual surcharge of 7 basis points compared to the evolving 41%/73% glide path, see *Exhibit 3*.

Exhibit 3

Given equity allocation										
Year 1	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%
Year 2	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%
Mean	0.127	0.139	0.152	0.164	0.177	0.190	0.202	0.215	0.228	0.241
Volatility	0.117	0.118	0.129	0.148	0.173	0.201	0.232	0.265	0.299	0.334
Optimal equity allocation										
Year 1	8%	9%	17%	25%	33%	41%	49%	58%	73%	87%
Year 2	9%	12%	27%	42%	58%	73%	88%	100%	100%	100%
Mean	0.137	0.140	0.153	0.166	0.179	0.192	0.205	0.218	0.231	0.242
Volatility	0.117	0.118	0.129	0.148	0.173	0.201	0.232	0.265	0.299	0.334
"Surcharge" (bp)	31.8	0.5	2.3	3.9	5.5	7.0	8.5	9.7	8.4	4.8

These "surcharges" are not only significant, but also unnecessary and avoidable. Overall, *Exhibits 1 - 3* clearly demonstrate that optimal glide paths are evolving.

Here are several additional observations regarding this example.

- The "surcharges" presented in *Exhibit 3* represent a "free lunch" readily available to the investor. This "free lunch" differs fundamentally from the classic MPT "free lunch" produced by optimal portfolio *selection*. In contrast, this "free lunch" is produced by optimal portfolio *evolution*.
- Optimal glide paths extend from the present to the end of the time horizon. It is assumed that the investor will promptly follow the instructions for future portfolio selections given today. This is the essence of "will-do" glide paths.
- Equity allocations are increasing for all optimal glide paths in this example. Yet, increasing equity allocations are not a general rule. Optimal glide paths may have different shapes for different objectives and assumptions.
- The "surcharges" presented in *Exhibit 3* are calculated to match the terminal value means generated by the optimal glide paths with reduced returns and the given glide paths with unreduced returns. Thus, these "surcharges" are based on the selection of the terminal value mean as the "matching" measurement. In general, the magnitude of such "surcharges" materially depends on the "matching" measurement. A different "matching" measurement is likely to generate different "surcharges."
- Optimal glide paths evolve regardless of the presence of "human capital."

Conclusion

As demonstrated in this series, certain optimal glide path design frameworks generate evolving glide paths. Other frameworks may generate stationary glide paths (e.g. Samuelson [1969]). Therefore, the structure of glide path design frameworks – objectives, assumptions, etc., – should be the center of attention. Mindlin [2015 A], Mindlin [2015 B], Mindlin [2015 C], this article and our other publications follow this principle.

Once again, why should glide paths evolve? To answer this question, one should present a sensible glide path framework in which optimal glide paths are generally evolving. The answer to this question presented in Mindlin [2015 C] is that the principle "any sub-glide path of an optimal glide path should be optimal on its own" generally leads to evolving glide paths. The answer to this question presented in this article is that the MPT-like mean-variance glide path optimization generally leads to evolving glide paths as well.

Overall, "expected-to-do" and "will-do" optimal glide paths are generally evolving. Future publications will explore these issues in more detail.

APPENDIX I: The Mean and Volatility of Terminal Values

Investor's Problem. Investor has $2 today and a commitment to make two payments: $1 in one year and $1 in two years.

r_i is portfolio *return* in year *i*, $R_i \equiv 1 + r_i$ is portfolio *return factor* in year *i*, *i*=1,2.

Terminal value (*TV*) for this problem is defined as follows:

$$TV = 2R_1R_2 - R_2 - 1$$

The mean of *TV* is calculated as follows:

$$E(TV) = 2E(R_1)E(R_2) - E(R_2) - 1$$

The volatility (standard deviation) of *TV* is calculated as follows:

$$StDev(TV) = \sqrt{4\left(Var(R_1)E(R_2)^2 + Var(R_2)E(R_1)^2 + Var(R_1)Var(R_2)\right) - Var(R_2)(4E(R_1) - 1)}$$

$E(R_i)$ and $Var(R_i)$, *i*=1,2, are calculated directly from the capital market assumptions.

APPENDIX II: Capital Market Assumptions

Return/Risk

	Geometric Mean (%)	Arithmetic Mean (%)	Standard Deviation (%)
Stocks	7.00	8.03	16.00
Bonds	4.00	4.12	5.00

Correlation Matrix

	Stocks	Bonds
Stocks	1	0.2
Bonds	0.2	1

Lognormal distributions are used to approximate portfolio returns.

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